**Müəllimin adı**: Samir Quliyev

**Fənnin adı**: Linear Algebra and Calculus

**Qrupun nömrəsi**: 1063

**Mid-term Exam I**

**Mövzu 1**: Functions and Graphs

**1**. Find the slope of a line passing through the given two points

**2**. A wire \_\_ inches long is to be cut into four pieces to form a rectangle whose shortest side has a length of .

(a) Write the area of the rectangle as a function of ;

(b) Determine the domain of the function and graph the function over that domain;

(c) Use the graph of the function to approximate the maximum area of the rectangle. Make a conjecture about the dimensions that yield a maximum area.

**3**. Find the distance between the given point and the line

**4**. A line is represented by the equation:

(a) When is the line parallel to the -axis?

(b) When is the line parallel to the -axis?

(c) Give values for and such that the line has a given slope.

(d) Give values for and such that the line is perpendicular to the given line

(e) Give values for and such that the line coincides with the graph of the given line

**5**. Consider the graph of the function shown below.

\_\_\_

Use this graph to sketch the graphs of the following functions:

(a) (b) (c) (d)

(e) (f) (g)

**6**. The table shows the populations (in thousands) of the Azerbaijan Republic for the period 1960 through 2010. The variable represents the time in years:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1960 | 1970 | 1980 | 1990 | 2000 | 2010 |
|  | \_\_\_ | \_\_\_ | \_\_\_ | \_\_\_ | \_\_\_ | \_\_\_ |

Plot the data by hand and connect adjacent points with a line segment.

(a) Determine the year when the population increased least rapidly.

(b) Determine the year when the population increased most rapidly.

(c) Find the average rate of change of the population of the Azerbaijan Republic from 1960 through 2010.

(d) Use the average rate of change of the population to predict the population of the Azerbaijan Republic in 2016. (the actual number is 9747.0)

**Mövzu 2**: Systems of Linear Equations

**1**. Consider the system of linear equations

(a) Without doing any row operations on this set of equations, explain why this system is consistent.

(b) Find a non-trivial solution of this system if there is any.

**2**. Consider the following system

(a) Find values of *a* and *b* for which the resulting system has a unique solution.

(b) Find values of *a* and *b* for which the resulting system has an infinite number of solutions.

(c) Find values of *a* and *b* for which the resulting system has no solution.

(d) Graph the resulting lines for each of the systems in parts (a), (b), and (c).

**3**. Solve the following system of linear equations using either Gaussian or Gauss-Jordan elimination procedure

**4**. Determine the value(s) of such that the following system of linear equations has exactly one solution

**5**. Determine the value(s) of such that the following system of linear equations does not have a unique solution

**6**. Express the column as a linear combination of the columns of matrix if

and

**Mövzu 3**: Matrices and Matrix Operations

**1**. Solve the following three systems of linear equations, all of which have the same matrix of coefficients

for

Which method of solution is more advantageous for this kind of systems of linear equations? (Explain your reasoning)

**2**. Let

and .

(a) Calculate and .

(b) Make a conjecture about the transpose of a product of two square matrices.

**3**. Determine the inverse of the matrix by solving a set of systems of linear equations

**4**. The first two test scores for Arzu, Hamid, Cafar, and Leyla are shown in the following table:

|  |  |  |
| --- | --- | --- |
|  | Test 1 | Test 2 |
| Arzu | \_ | \_ |
| Hamid | \_ | \_ |
| Cafar | \_ | \_ |
| Leyla | \_ | \_ |

Use the table to create a matrix to represent the data. Use this matrix to answer the following questions.

(a) Which test was more difficult? (Give an explanation)

(b) Which test was easier? (Give an explanation)

(c) Describe the meaning of the matrix products and .

(d) Describe the meaning of the matrix products and .

**5**. The first two test scores for Aydin, Sardar, Ceyran, and Samad are shown in the following table:

|  |  |  |
| --- | --- | --- |
|  | Test 1 | Test 2 |
| Aydin | \_ | \_ |
| Sardar | \_ | \_ |
| Ceyran | \_ | \_ |
| Samad | \_ | \_ |

Use the table to create a matrix to represent the data. Use this matrix to answer the following questions.

(a) Describe the meaning of the matrix products and .

(b) Describe the meaning of the matrix products and .

(c) Describe the meaning of the matrix product .

(d) Use matrix multiplication to express the combined overall average score on both tests.

(e) As the instructor, you would like to raise the scores on test 1 for all the students. How could you use matrix multiplication to scale the scores by a factor of 1.1?

**6.** Consider the system of homogeneous linear equations

(a) Show that if is a solution then is also a solution, for any value of the constant .

(b) Show that if and are any two solutions, then is also a solution.

**Mövzu 4**: Determinants

**1**. Determine the minors and cofactors of the following matrix

**2**. Find the determinant of the matrix using expansion by cofactors

**3**. Find the determinant of the matrix using elementary row or column operations

**4**. Solve the following equation for

**5**. Let and be square matrices of order 4:

Find (a) (b) (c) (d) (e)

**Mövzu 5**: Eigenvalues and Eigenvectors

**1**. Find the eigenvalues of the matrix

**2**. Find the eigenvalues and eigenvectors of the matrix

**Mid-tem Exam II**

**Mövzu 6**: Linear Inequalities

**1**. Two types of tickets are to be sold for a concert. One type costs \_AZN per ticket and the other type costs \_AZN per ticket. The promoter of the concert must sell at least \_ tickets including \_ of the \_AZN tickets and \_ of the \_AZN tickets. Moreover, the gross receipts must total at least \_AZN in order for the concert to be held. Find a system of inequalities describing the different numbers of tickets that can be sold, and sketch the graph of the system.

**2**. Sketch the graph of the solution of the system of linear inequalities:

**3**. Derive a set of inequalities to describe the region

(a)

(b)

**4**. A furniture company can sell all the tables and chairs it produces. Each table requires hour in the assembly center and hours in the finishing center. Each chair requires hours in the assembly center and hours in the finishing center. The company’s assembly center is available hours per day, and its finishing center is available hours per day. If is the number of tables produced per day and is the number of chairs, find a system of inequalities describing all possible production levels. Sketch the graph of the system.

**5**. A person plans to invest no more than $\_ in two different interest-bearing accounts. Each account is to contain at least $\_. Moreover, one account should have at least twice the amount that is in the other account. Find a system of inequalities to describe the various amounts that can be deposited in each account, and sketch the graph of the system.

**Mövzu 7**: Geometric Method for Solving Linear Programming Problem

**1**. A merchant plans to sell two models of home computers at costs of $\_ and $\_, respectively. The $\_ model yields a profit of $\_ and the $\_ model yields a profit of $\_. The merchant estimates that the total monthly demand will not exceed \_ units. Find the number of units of each model that should be stocked in order to maximize profit. Assume that the merchant does not want to invest more than $\_ in computer inventory.

**2**. A company makes two types of microcomputers, the Jupiter and the Cosmos. It takes \_ hours to assemble a Jupiter microcomputer, and it takes \_ hours to assemble a Cosmos microcomputer. The total labor time available for this work is \_ hours per week. The manufacturing cost of each Jupiter microcomputer is $\_ and the manufacturing cost of each Cosmos microcomputer is $\_. The total funds available per week for manufacturing are $\_. The profit on each Jupiter microcomputer is $\_, and the profit on each Cosmos microcomputer is $\_. How many of each type of microcomputer should be assembled weekly to obtain maximum profit?

**3**. Find the minimum and maximum values of the objective function, subject to the indicated constraints.

Objective function:

Constraints:

**4**. A fruit grower has \_ acres of land available to raise two crops, A and B. It takes one day to trim an acre of crop A and two days to trim an acre of crop B, and there are \_ days per year available for trimming. It takes \_ day to pick an acre of crop A and \_ day to pick an acre of crop B, and there are \_ days per year available for picking. Find the number of acres of each fruit that should be planted to maximize profit, assuming that the profit is $\_ per acre for crop A and $\_ per acre for crop B.

**5**. The given linear programming problem has an unusual characteristic. Sketch a graph of the solution region for the problem and describe the unusual characteristic. (In each problem, the objective function is to be maximized.):

**6**. Determine -values such that the objective function has a maximum value at the indicated vertex:

(a) (b) (c) (d)

**Mövzu 8**: Simplex Method for Solving Linear Programming Problem

**1**. Use the simplex method to find the maximum value of

subject to the constraints

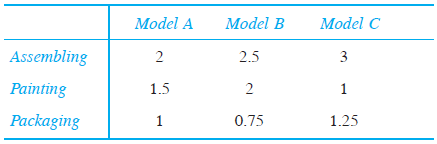
where .

**2**. Use the simplex method to find the maximum value of

subject to the constraints

where .

**3**. A manufacturer produces three models of bicycles. The time (in hours) required for assembling, painting, and packaging each model is as follows:



The total time available for assembling, painting, and packaging is \_ hours, \_ hours, and \_ hours, respectively. The profit per unit for each model is $\_ (Model A), $\_ (Model B), and $\_ (Model C). How many of each type should be produced to obtain a maximum profit?

**4**. Find the minimum value of

subject to the constraints

where .

**Mövzu 9**: Limit of a Function

**1**. Find the limit of the function

**2**. Estimate the limit of the function

**3**. Determine the limit of the trigonometric function

**4**. Find the values of the constants and such that

**5**. Determine all values of the constant such that the following function is continuous for all real numbers

**Mövzu 10**: Limits and the Derivative

**1**. Find the point on the graph of (see figure) where the tangent line has the greatest slope, and the point where the tangent line has the least slope.

“Here comes the figure of the graph of ”

**2**. Find the dimensions of the rectangle of maximum area, with sides parallel to the coordinate axes, that can be inscribed in the ellipse given by

**3**. Analyze (symmetricity, and intercepts, vertical asymptotes, horizontal asymptotes, etc.) and sketch the graph of the function .

**4**. Find the limit at infinity of the function

**5**. Find an equation of the line that is tangent to the graph of and parallel to the given line

, Line:

**Final Exam**

**Mövzu 11**: Applications of Differentiation

**1**. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground. How many seconds after its release will the bag strike the ground?

**2**. Find a point on the interval , at which the instantaneous rate of change of the function is equal to its average rate of change over the entire interval .

**3**. Explain why the function has a zero in the interval . Approximate the zero of the function in this interval.

**4**. Neglecting air resistance, the path of a projectile that is propelled at an angle is

Here is the height, is the horizontal distance, is the acceleration due to gravity, is the initial velocity, and is the initial height. Let feet per second per second, feet per second, and feet. What value of will produce a maximum horizontal distance?

**5**. With what initial velocity must an object be thrown upward (from ground level) to reach the top of the Washington Monument (approximately 550 feet)?

**Mövzu 12**: Graphing and Optimization

**1**. Find the maximum and minimum points on the graph of the implicit function

1. using Precalculus techniques
2. using Calculus techniques

**2**. Approximate any relative extrema and asymptotes (both vertical and horizontal) of the function

**3**. Find the critical numbers (if any), the open intervals on which the function is increasing or decreasing, and the open intervals on which the function is concave downward and concave upward

**4**. The graph of the function consists of the three line segments joining the points , , , and . The function is defined by the integral

Find the extrema of on the given interval.

**5**. Find a polynomial that fits the given five points

**Mövzu 13**: Integration

**1**. Find the area of the region between the graph of the function and the -axis over the given interval:

**2**. Evaluate the definite integral of the algebraic function:

**3**. Find the average value of the function over the given interval and all values of in the interval for which the function equals its average value:

**4**. Suppose that is integrable on and for all in the interval . Prove that

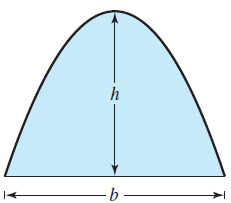
**5**. A particle is moving along a line so that its velocity is feet per second at time .

(a) What is the displacement of the particle on the time interval ?

(b) What is the total distance traveled by the particle on the time interval ?

**Mövzu 14**: Applications of Integration

**1**. Archimedes showed that the area of a parabolic arch is equal to the product of the base and the height. Prove Archimedes’ formula for a general parabola.



**2**. Find the least-squares approximation for the function

**3**. Find a solution of the differential equation. Then find an appropriate interval of definition of the solution

**4**. Determine

by using an appropriate Riemann sum.

**5**. Solve the system of linear differential equations

**Mövzu 15**: Taylor Polynomials and Approximations

**1**. Use a power series to approximate the definite integral

with an error of less than 0.01.

**2**. Find the power series for

**3**. Find the polynomial whose value and first three derivatives agree with the value and first three derivatives of at the point . This polynomial is called the third-degree “Taylor polynomial” of at the point .

**4**. Find the polynomial whose value and first two derivatives agree with the value and first two derivatives of at the point . This polynomial is called the second-degree “Taylor polynomial” of at the point .

**5**. Estimate the given value by using the Taylor expansion polynomial of the function at the point .