

**AZƏRBAYCAN RESPUBLİKASI TƏHSİL NAZİRLİYİ**

**AZƏRBAYCAN DÖVLƏT İQTİSAD UNİVERSİTETİ**

**BEYNƏLXALQ İQTİSADİYYAT MƏKTƏBİ**

**Final Exam – Tests for Students**

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**Fənnin adı**: Linear Algebra and Calculus

**Qrupun nömrəsi**: 1064

**Mövzu 1**: Systems of Linear Equations

**1.1**. Consider the system of linear equations

(**a**) Without doing any row operations on this set of equations, explain why this system is consistent.

(**b**) Find a non-trivial solution of this system in the parametric form if there is any.

**1.2**. Consider the following system

(**a**) Find values of *a* and *b* for which the resulting system has a unique solution.

(**b**) Find values of *a* and *b* for which the resulting system has an infinite number of solutions.

(**c**) Find values of *a* and *b* for which the resulting system has no solution.

(**d**) Graph the resulting lines for each of the systems in parts (a), (b), and (c).

**1.3**. Solve the following system of linear equations using either Gaussian or Gauss-Jordan elimination procedure

**1.4**. Determine the value(s) of such that the following system of linear equations

**(a**) has exactly one solution

(**b**) has no solution

(**c**) has an infinite number of solutions

**1.5**. Determine the value(s) of such that the following system of linear equations

(**a**) has exactly one solution

(**b**) has no solution

(**c**) has an infinite number of solutions

**1.6**. Express the column as a linear combination of the columns of matrix if

 and

**1.7**. Solve (if possible) the following system of linear equations using the Gauss-Jordan elimination procedure:

**Mövzu 2**: Matrices

**2.1**. Let be a matrix, a matrix, a matrix, a matrix, a column matrix. Determine which of the following matrix expressions exist, and give the size of the resulting matrices when they do exist:

(**a**) (**b**) (**c**) (**d**) (**e**)

(**f**) (**g**)

**2.2**. The first two test scores for Arzu, Hamid, Cafar, and Leyla are shown in the following table:

|  |  |  |
| --- | --- | --- |
|  | Test 1 | Test 2 |
| Arzu | 64 | 76 |
| Hamid | 86 | 92 |
| Cafar | 78 | 73 |
| Leyla | 82 | 81 |

Use the table to create a matrix to represent the data. Use this matrix to answer the following questions.

(**a**) Which test was more difficult? (Give an explanation)

(**b**) Which test was easier? (Give an explanation)

(**c**) Describe the meaning of the matrix products and .

(**d**) Describe the meaning of the matrix products and .

**2.3**. The first two test scores for Aydin, Sardar, Ceyran, and Samad are shown in the following table:

|  |  |  |
| --- | --- | --- |
|  | Test 1 | Test 2 |
| Aydin | 64 | 76 |
| Sardar | 86 | 92 |
| Ceyran | 78 | 73 |
| Samad | 82 | 81 |

Use the table to create a matrix to represent the data. Use this matrix to answer the following questions.

(**a**) Describe the meaning of the matrix products and .

(**b**) Describe the meaning of the matrix products and .

(**c**) Describe the meaning of the matrix product .

(**d**) Use matrix multiplication to express the combined overall average score on both tests.

(**e**) As the instructor, you would like to raise the scores on test 1 for all the students. How could you use matrix multiplication to scale the scores by a factor of 1.1?

**2.4.** Consider the system of homogeneous linear equations

(**a**) Show that if is a solution then is also a solution, for any value of the constant .

(**b**) Show that if and are any two solutions, then is also a solution.

**2.5**. Determine the equation of the polynomial of degree 2, , whose graph passes through the points .

**Mövzu 3**: Matrix Operations

**3.1**. Let

 and .

(**a**) Calculate and .

(**b**) Make a conjecture about the transpose of a product of two square matrices.

**3.2**. If is a square matrix, prove that

(**a**) is symmetric;

(**b**) is antisymmetric (Definition: a matrix is said to be antisymmetric if ).

**3.3**. Prove the following properties:

(**a**) If and are square matrices of the same size, prove that in general

Under what conditions does equality hold?

(**b**) If and are square matrices of the same size such that , prove that . By constructing an example, show that this result does not hold for all square matrices of the same size.

**Mövzu 4**: Inverse Matrices

**4.1**. Solve the following three systems of linear equations, all of which have the same matrix of coefficients

 for

Which method of solution is more advantageous for this kind of systems of linear equations? (Explain your reasoning)

**4.2**. Determine the inverse of the matrix (if it exists)

**4.3**. Use the matrix inverse method to solve the following system of linear equations:

**Mövzu 5**: Determinants

**5.1**. Determine the minors and cofactors of the following matrix

**5.2**. Find the determinant of the matrix using expansion by cofactors

**5.3**. Find the determinant of the matrix using elementary row or column operations

**5.4**. Solve the following equation for

**5.5**. Let and be square matrices of order 4:

Find:

(**a**) (**b**) (**c**) (**d**) (**e**)

**5.6.** Find the value(s) of such that is singular:

**5.7.** Find the eigenvalues of the matrix

**Mövzu 6**: Geometric Method for Solving Linear Programming Problems

**1**. A merchant plans to sell two models of home computers at costs of $280 and $380, respectively. The $280 model yields a profit of $45 and the $380 model yields a profit of $50. The merchant estimates that the total monthly demand will not exceed 230 units. Find the number of units of each model that should be stocked in order to maximize profit. Assume that the merchant does not want to invest more than $60000 in computer inventory.

**2**. A company makes two types of microcomputers, the Jupiter and the Cosmos. It takes 5 hours to assemble a Jupiter microcomputer, and it takes 2 hours to assemble a Cosmos microcomputer. The total labor time available for this work is 1000 hours per week. The manufacturing cost of each Jupiter microcomputer is $6 and the manufacturing cost of each Cosmos microcomputer is $8. The total funds available per week for manufacturing are $3000. The profit on each Jupiter microcomputer is $4, and the profit on each Cosmos microcomputer is $4. How many of each type of microcomputer should be assembled weekly to obtain maximum profit?

**3**. Derive a set of inequalities to describe the region

(a)

(b)

**4**. A fruit grower has 150 acres of land available to raise two crops, A and B. It takes one day to trim an acre of crop A and two days to trim an acre of crop B, and there are 240 days per year available for trimming. It takes 0.4 day to pick an acre of crop A and 0.2 day to pick an acre of crop B, and there are 30 days per year available for picking. Find the number of acres of each fruit that should be planted to maximize profit, assuming that the profit is $150 per acre for crop A and $230 per acre for crop B.

**5**. The given linear programming problem has an unusual characteristic.

Sketch a graph of the solution region for the problem and describe the unusual characteristic (in the problem, the objective function is to be maximized).

**6**. Determine -values such that the objective function has a maximum value at the indicated vertex:

(a) (b) (c) (d)

**Mövzu 7**: Simplex Method for Solving Linear Programming Problems

**1**. Use the simplex method to find the maximum value of

subject to the constraints

where .

**2**. Use the simplex method to find the maximum value of

subject to the constraints

where .

**3**. A manufacturer produces three models of bicycles. The time (in hours) required for assembling, painting, and packaging each model is as follows:



The total time available for assembling, painting, and packaging is 4000 hours, 2400 hours, and 1600 hours, respectively. The profit per unit for each model is $40 (Model A), $60 (Model B), and $50 (Model C). How many of each type should be produced to obtain a maximum profit?

**4**. Find the minimum value of

subject to the constraints

where .

**Mövzu 8**: Functions and Graphs

**1**. Find the distance between the point and the line

**2**. Find the distance between the two lines

Line 1: Line 2:

**3**. Consider the graph of the function . Use this graph to sketch the graphs of the following functions:

(a) (b) (c) (d)

(e) (f) (g)

**4**. You are in a boat 2 miles from the nearest point on the coast. You are to go to a point located 3 miles down the coast and 1 mile inland (see Figure).



You can row at 2 miles per hour and walk at 4 miles per hour. Write the total Time of the trip as a function of .

**5.** Consider the shaded region outside the sector of a circle of radius 10 meters and inside a right triangle



Write the perimeter of the region as a function of .

**Mövzu 9**: Limit of a Function

**1**. Find the limit of the function

**2**. Estimate the limit of the function

**3**. Determine the limit of the trigonometric function

**4**. Find the values of the constants and such that

**5**. Determine all values of the constant such that the following function is continuous for all real numbers

**Mövzu 10**: Limits and the Derivative

**1**. Find the point on the graph of where the tangent line has the greatest slope, and the point where the tangent line has the least slope.

**2**. Find the dimensions of the rectangle of maximum area, with sides parallel to the coordinate axes, that can be inscribed in the ellipse given by

**3**. Analyze (symmetricity, and intercepts, vertical asymptotes, horizontal asymptotes, etc.) and sketch the graph of the function

**4**. Find the limit at infinity of the function

**5**. Find an equation of the line that is tangent to the graph of and parallel to the given line

, Line:

**Mövzu 11**: Applications of Differentiation

**1**. (a) A balloon, rising vertically with a velocity of 10 feet per second, releases a sandbag at the instant it is 50 feet above the ground. How many seconds after its release will the bag strike the ground?

(b) With what initial velocity must an object be thrown upward (from ground level) to reach the top of the Washington Monument (approximately 500 feet)?

**2**. Find a point on the interval , at which the instantaneous rate of change of the function is equal to its average rate of change over the entire interval .

**3**. Explain why the function has a zero in the interval . Approximate the zero of the function in this interval (using either the Newton’s method or the bisection method).

**4**. Find the critical numbers (if any), the open intervals on which the function is increasing or decreasing, and the open intervals on which the function is concave downward and concave upward:

**5**. Two stationary patrol cars equipped with radar are 6 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 51 miles per hour. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 52 miles per hour. Prove that the truck must have exceeded the speed limit (of 55 miles per hour) at some time during the 5 minutes.

**Mövzu 12**: Graphing and Optimization

**1**. Find the maximum and minimum points on the graph of the implicit function

1. using Precalculus techniques
2. using Calculus techniques

**2**. Approximate any relative extrema and asymptotes (both vertical and horizontal) of the function

**3**. (a) A rectangle is bounded by the axis and the semicircle . What length and width should the rectangle have so that its area is a maximum?



(b) Find the dimensions of the largest rectangle that can be inscribed in a semicircle of radius.

**4**. The graph of the function consists of the three line segments joining the points , , , and . The function is defined by the integral

Find the extrema of on the interval .

**5**. Find a polynomial function , that has only the specified extrema: relative minima at and ; relative maximum at . Determine the minimum degree of the function and give the criteria you used in determining the degree.

**Mövzu 13**: Integration

**1**. Find the area of the region between the graph of the function and the -axis over the given interval:

**2**. Evaluate the definite integral of the algebraic function:

**3**. Find the average value of the function over the given interval and all values of in the interval for which the function equals its average value:

**4**. Find the area of the unbounded shaded region:

(a) (b)



**5**. A particle is moving along a line so that its velocity is

feet per second at time .

(a) What is the displacement of the particle on the time interval ?

(b) What is the total distance traveled by the particle on the time interval ?

**Mövzu 14**: Applications of Integration

**1**. Archimedes showed that the area of a parabolic arch is equal to the product of the base and the height. Prove Archimedes’ formula for a general parabola.



**2**. Find the least-squares approximation for the function

**3**. Determine the limits of integration where such that

has minimal value.

**4**. Determine

by using an appropriate Riemann sum.

**5**. Suppose that is integrable on and for all in the interval . Prove that

Use this result to estimate the definite integral

**Mövzu 15**: Taylor Polynomials and Approximations

**1**. Use a power series to approximate the definite integral

with an error of less than 0.001.

**2**. Find the power series for

**3**. Find the polynomial whose value and first three derivatives agree with the value and first three derivatives of at the point . This polynomial is called the third-degree “Taylor polynomial” of at the point .

**4**. Find the polynomial whose value and first two derivatives agree with the value and first two derivatives of at the point . This polynomial is called the second-degree “Taylor polynomial” of at the point .

**5**. Estimate the value by using the Taylor expansion polynomial of the function at the point .