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Inverse problem of contact fracture mechanics for a hub of friction pair taking into account thermal stresses

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Abstract
Methods of fracture mechanics enable a new approach to the design of structures, ensuring prevention of crack development. The plane problem of mechanics of contact fracture for the hub of a friction pair during operation is studied. It is accepted that near the rough friction surface, the hub has a rectilinear crack. A criterion and a method for solving the inverse problem of the mechanics of contact fracture on definition of the function of displacements of external contour points of the hub of a friction pair with regard to temperature drop and inequalities of the contact surface in friction pair components is given. The found function of displacements of the external contour points of the hub provides an increase of the load-bearing capacity of the hub of a friction pair. The problem of prevention of the fracture of the hub of a friction pair with allowance for the real friction surface was first posed and then solved.

Keywords
Friction pair, temperature, rough friction surface, thermal stresses, closure of crack faces, function of displacements of external contour of hub

1. Introduction

The life of a friction pair [1,2] is determined by the efficiency of the hub and the stress distribution in the zones of interaction of the friction pair details. The practice of exploitation of the friction pairs of oil-field equipment and of transport vehicles shows that at repeated reciprocating motion of the plunger, fracture of the hub of a friction pair occurs on the spots of actual touch in thin surface layers by micro cracking, within which the hub "lives" the substantial part of its life. In connection with this, at the stage of design of new constructions of movable conjugations, it is necessary to carry out limit analysis of contact pair details to establish that the would-be initial cracks allocated adversely will not grow to critical sizes and will not cause fracture during its design life. The size of the initial minimal crack should be considered as a design characteristic of the material.

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It should be taken into account that the bushing internal contour and the plunger external contour are nearly circular. As is known, real treated surfaces are never absolutely smooth, but always has micro- or macroscopic irregularities (of technological character) forming the rough surface. Despite the extremely small sizes of such irregularities, they affect the different service properties of tribo-conjugation [3–6].

In the contemporary stage of engineering the optimal design of the friction pair, details providing an increase of efficiency of friction pairs are of great value [7–31]. The theoretical analysis on the determination of interference of the jointed cylinder, providing minimization of the abrasive wear of the friction unit, was carried out by Gadzhiev and Mirsalimov [7]. In Gadzhiev and Mirsalimov [8], the interference fit of the jointed cylinder, providing the absence of initial wear, was determined. The model of the friction rough surface was used. In Peigney [9], a method was developed to determine the asymptotic state reached by a solid continuum subjected to wear and submitted to acyclic loading. The main idea is to express the stabilized state as the solution of a minimization problem. In Mirsalimov [10,11], the problems of minimization of stress and the thermal state of contact pair bushing were investigated. Chu [12] aimed to develop an algorithm for designing the optimum shape of the slider bearing and pressure distribution using an inverse method. Based on a model of a rough friction surface, the microgeometry of a friction surface that ensures a uniform distribution of contact pressure is theoretically analyzed by Mirsalimov [13]. A plane problem of fracture mechanics for a circular cracked disk fitted onto a rotating shaft is considered by Mirsalimov [14]. The interference between the disk and the shaft, providing minimization of fracture parameters (stress intensity factors) of the disk, is studied. Chu et al. [15] aimed to develop an algorithm for designing the piston ring profile and pressure distribution using an inverse method. Alyaqout and Elsharkawy [16] introduced an approach for designing the optimum shape of a slider bearing using an inverse method. The proposed approach utilizes a sequential quadratic programming algorithm to minimize friction subject to the load and center of pressure requirements specified. In Mirsalimov [17], the minimax criterion is used to determine the microgeometry of friction surfaces to achieve the minimal contact pressure in the bush–plunger friction pair. According to the lubrication theory and the reliability design theory, the optimum design of the hyperboloidal sliding bearing based on the reliability constraints was put forward by Xu et al. [18]. In Huang et al. [19], the optimal clearance formula of the piston friction pair for a water hydraulic pump was established based on the thermal balance principle. Simulations and experimental research were carried out on its influence factors; the results show that a minimum thermal clearance exists in each of the piston friction pair and its initial clearance must be greater than this one, otherwise it will be destroyed. Hussain and Sonpimple [20] describe the development of the evolutionary algorithm for the optimal design of the friction clutch. The Genetic Algorithm is applied for the optimum design of the friction clutch. The main objective function was to minimize the axial force between the clutch plate and the pressure plate, using the diameter ratio and the coefficient of friction as design variables and the peripheral velocity as the design constraint. Theoretical analysis of the definition of the negative allowance providing minimization of failure parameters of a drawing die reinforced with a holder was carried out on the minimax criterion by Mirsalimov and Veliyev [21]. Paredes et al. [22] define a method for the optimization of design parameter tolerances. The general architecture of the proposed method is identical to that of the robust design reference method proposed by Taguchi, but its content is different as the tolerances are considered as functions to be maximized. The possibilities offered by this method are illustrated through its use in the preliminary design of a cold-expanded bushing. Based on the minimax criterion, the theoretical definition of the displacement function for hub external contour points, providing the minimization of contact pressure in the hub–shaft friction pair, is executed by Mirsalimov and Akhundova [23]. The generalized cumulative model of wear is employed to solve the tribocontact problem for plain bearings where journals have small lobing (ovality, trilobing, tetralobing) in paper [24]. The case of mixed-area contact interaction between the shaft and the bush is examined. It was found that lobing causes a significant increase in the contact pressures, depending on the extent of shaft out-of-roundedness and the type of lobing. To enhance the wear resistance of the star-wheel, an approach for optimization of the star-wheel profile was developed by Huang et al. [25]. In Abdo [26], a mathematical model is developed to correlate the volumetric wear of materials with the dissipation energy in sliding contacts. The wear of contacting materials originating from the energy loss due to the friction process in the contact is studied. To reduce the friction of a piston ring, an approach comprising the inverse method and the sequential quadratic programming algorithm was proposed by Zhang et al. [27]. In Mirsalimov and Akhundova [28], based on the minimax criterion and the model of the friction rough surface, a theoretical definition is developed for the displacement function of the hub external contour that provides the minimization of abrasive wear in the hub–plunger friction pair. Zhang et al. [29] aimed at proposing an approach for optimizing the shape of the top piston ring face for minimum friction force using an inverse method. In Mirsalimov and Akhundova [30], the theoretical analysis of the displacement function for external contour points of the hub of a frictional pair, providing the absence of the initial wear and tear, which takes place toward the end of the bedding period, is carried out. A criterion and a method for solving the inverse problem of fracture mechanics on definition of the function of displacements of the external contour points of the hub of a friction pair with regard to temperature drop in the friction pair are given by Mirsalimov and Akhundova [31].

In the above studies on the optimization of friction pairs, the presence of a crack in the hub of the friction pair was not taken into account. In Mirsalimov and Akhundova [31], the problem of the optimization of friction pairs in the presence of a rectilinear crack in the hub is first considered in a simplified formulation. In Mirsalimov and Akhundova [31] it is assumed that the friction surface is absolutely smooth. In fact, the real friction surface is rough and contains technological irregularities. In the present work, in contrast to Mirsalimov and Akhundova [31], it is taken into account that the friction surface is rough.

The solution of a mechanics problem on the definition of such a displacement function of the external contour points of the hub at which the stress field formed by it would retard the crack propagation in the hub, is of considerable interest.

2. Formulation of the problem

Let us consider the stress–strain state of the hub of a friction pair. During operation of the friction pair at repeatedly reciprocating motion of the plunger, there happens force interaction between the contacting surfaces of the hub and the plunger, and there arise friction forces that reduce the wear of the conjugation materials. To determine the contact pressure, it is necessary to consider [1,2] a wear-contact problem on pressing of a plunger into the surface of the hub weakened by a rectilinear crack.

Let a plunger with elastic parameters G_1 and μ_1 be pressed to the internal surface of the hub with elastic parameters G (shear modulus) and μ (Poisson ratio) at some beforehand unknown area. It is considered that on the external contour, the hub has some displacements. The function of these displacements is not known in advance, and should be determined from the additional condition.

It is accepted that the plane strain conditions are fulfilled. The modes of the friction pair at which there may arise residual deformations are unacceptable. An elastic hub has a rectilinear crack of length $2\ell_1$ (Figure 1). Refer the hub of the friction pair to the polar system of coordinates $r\theta$ having chosen the origin of the coordinates in the center of concentric circle L of radii R (Figure 1). We will assume that the internal contour of the hub and the external contour of the plunger are close to circular.

Represent the boundary of the internal contours of the hub L' in the form $r = \rho(\theta)$; $\rho = R + \varepsilon H(\theta)$, where $\varepsilon = R_{\max}/R$ is a small parameter; $R_{\max}$ is the greatest height of the irregularity of the friction surface.

By means of a profilogram of the processed surface of the hub, we find the coefficients of the Fourier series for the function $H(\theta)$ describing each inner profile of the hub

$$H(\theta) = \sum_{k=0}^{n} \left(a_k^0 \cos k\theta + b_k^0 \sin k\theta\right).$$

The outer contour of the plunger is nearly circular and may be represented as

$$\rho_1(\theta) = R' + \varepsilon H_1(\theta), H_1(\theta) = \sum_{k=0}^{n} \left(a_k^1 \cos k\theta + b_k^1 \sin k\theta\right).$$

It is assumed that the wear of the hub and the plunger is of abrasive character.

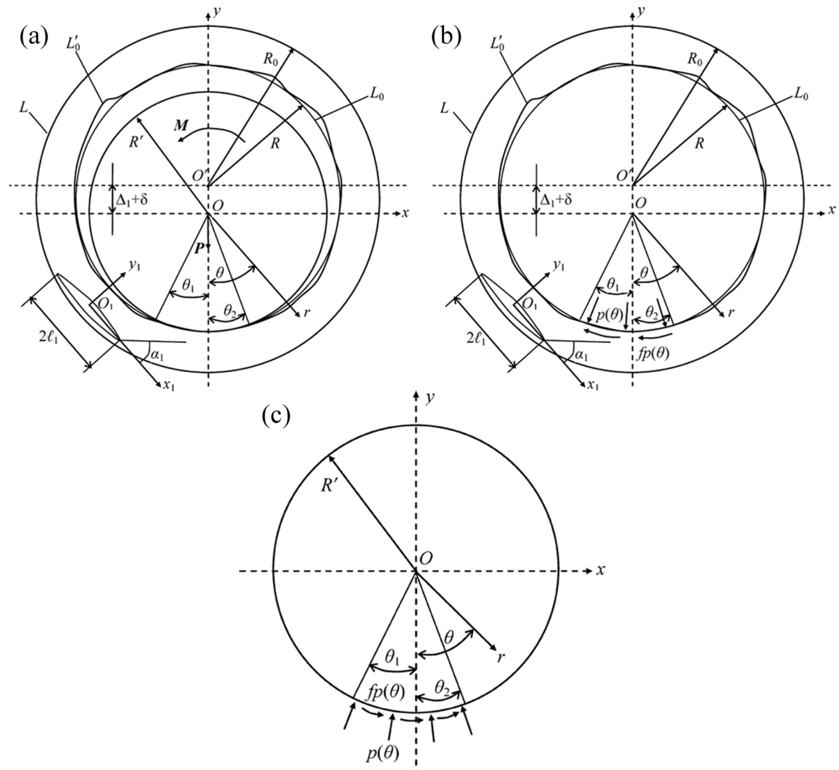

Figure 1. Design diagram of the contact problem for the hub–plunger friction pair: (a) friction pair loading diagram; (b) hub loading diagram; (c) plunger loading diagram.

In the center of the rectilinear crack we place the origin of the local system of coordinates $x_1O_1y_1$, whose axis x_1 coincides with the line of the crack and shapes the angle α_1, with the axis x (Figure 1). It is accepted that the crack faces are free from external loads. The condition relating the displacements of the hub and plunger is of the form [1,2]

$$v_1 + v_2 = \delta(\theta) \quad (\theta_1 \leqslant \theta \leqslant \theta_2). \tag{1}$$

Here $\delta(\theta)$ is the sag of the points of the surfaces of the hub and the plunger determined by the form of the internal surface of the hub and plunger, and also by the magnitude of the compressing force $\boldsymbol{P}$; θ_2–θ_1 is the size of the contact angle (area).

In the contact zone, in addition to contact pressures there acts the tangential stress $\tau_{r\theta}$ connected with the contact pressure $p(\theta,t)$ by the Amontons–Coulomb law

$$\tau_{r\theta} = fp(\theta, t),$$

where f is the friction coefficient of the pair "hub–plunger".

The tangential forces (friction forces) $\tau_{r\theta}(\theta,t)$ help to release heat in the contact zone. The total amount of heat per unit time is proportional to the power of friction forces, and the amount of heat released at the contact zone point with the coordinate θ will be equal to

$$Q(\theta, t) = Vfp(\theta, t),$$

where V is the mean rate for the period of displacements of the plunger.

The general amount of the heat $Q(\theta,t)$ will be consumed in the following way: heat flow to the hub $Q_b(\theta,t)$ and similar flow $Q_1(\theta,t)$ of heat to increase temperature of the plunger, that is, $Q = Q_b + Q_1$.

For displacements of the friction surface points of the hub, we have $v_1 = v_{1e} + v_{1r} + v_{1w}$, where v_{1e} are thermoelastic displacements of the contact surface points of the hub; v_{1r}, v_{1w} are displacements caused by crushing of microprojections and wear of the hub surface, respectively. Similarly, for displacements of the plunger contact surface, we have $v_2 = v_{2e} + v_{2r} + v_{2w}$.

The rate of change of displacements of the surface at wear of the hub and plunger will be equal to [1,2]

$$\frac{dv_{kw}}{dt} = K^{(k)} p(\theta, t) (k = 1, 2), \tag{2}$$

where $K^{(k)}$ are the coefficients of wear of the hub and plunger material ($k = 1,2$), respectively.

As the motion of frequency of the plunger is rather great, we consider the problem as a stationary one. In this case the hub temperature $T(r,\theta)$ satisfies the differential equation of heat conduction theory $\Delta T = 0$ and the boundary conditions

$$A_{T1}\lambda \frac{\partial T}{\partial n} - A_{T2}\alpha_1^*(T - T_c) = -Q_*(\theta) \quad \text{for } r = R,$$
$$\lambda \frac{\partial T}{\partial n} + \alpha_2(T - T_c) = 0 \quad \text{for } r = R_0.$$

Here λ is the coefficient of heat conductivity of the hub; Δ is the Laplace operator; α_1^* is the coefficient of heat transfer from the inner surface of the hub; α_2 is the coefficient of transfer from the external cylindrical surface of the hub with the external medium of temperature T_c; A_{T1} is the heat absorbing surface; A_{T2} is the cooling surface; Q_* is a part of the amount of heat released at friction necessary for heating the hub; $Q_* = Q_b$ on the contact area; $Q_* = 0$ out of the area; n, t are natural coordinates.

The perturbed temperature field caused by the crack is ignored in determining the temperature field to simplify the problem.

To determine the displacements v_{1e} and v_{1r}, it is necessary to solve a thermoelasticity problem for the hub under the following conditions

$$r = \rho(\theta), \sigma_n = -p(\theta), \tau_{nt} = -fp(\theta) \text{ on the contact area;}$$
$$\sigma_n = 0, \tau_{nt} = 0 \text{ out of the contact area for } r = \rho(\theta),$$
$$v_r - iv_\theta = g(\theta) \text{ for } r = R_0,$$
$$\text{on the crack faces } \sigma_{y_1} = 0, \tau_{x_1y_1} = 0.$$

Here σ_n, τ_{nt}, σ_{y_1}, $\tau_{x_1y_1}$ are the stress tensor components; v_r, v_θ are the radial and tangential components of the displacements vector of the contour L, respectively; $g(\theta)$ is the sought-for function of displacements of the points of the external contour L of the hub; $i^2 = -1$.

A thermoelasticity problem for determining the displacements v_{2e} and v_{2r} of the contact surface of the plunger is stated in the same way

$$\Delta T_1 = 0,$$

$$\lambda_1 \frac{\partial T_1}{\partial n} = -Q_1(\theta) \quad \text{on the contact area for } r = \rho_1(\theta),$$
$$\lambda_1 \frac{\partial T_1}{\partial n} + \alpha_1^* T_1 = 0 \quad \text{out of the contact area,}$$
$$\sigma_n = -p(\theta), \tau_{nt} = -fp(\theta) \quad \text{on the contact area for } r = \rho_1(\theta),$$
$$\sigma_n = 0, \tau_{nt} = 0 \quad \text{out of the contact area.}$$

The contact pressure $p(\theta)$ is unknown beforehand and should be determined in the course of solution of the contact fracture mechanics problems. The quantities θ_1 and θ_2, being the ends of the contact area of the plunger and hub, are unknown. For determining them, we use the condition [32] that the pressure $p(\theta)$ continuously goes to zero when the point θ goes beyond the contact section

$$p(\theta_1) = 0, p(\theta_2) = 0. \tag{3}$$

To find the sought-for function $g(\theta)$ of displacements of the points of the external contour L of the hub, the problem statement should be complemented with the condition (criterion) of definition of the function $g(\theta)$. As such, a condition of definition of the function of displacements of the points of the contour L (of the function $g(\theta)$), we accept that in the process of work of the friction pair at the crack tips there should appear end zones whose faces are closed, that is, come into contact. Closing of the crack faces at the end zones adjacent to the crack tips restrains the crack propagation and thereby retards the fracture process of the hub of the friction pair.

Then, it is required to determine the function of displacements of the points of the external contour L of the hub (of the function $g(\theta)$) so that the stress–strain field formed by it in the process of work of the friction pair could provide the closure of the crack faces in the end zones adjacent to the crack tips.

At the end zones where the closure of crack faces happens, the crack opening should vanish

$$\left(u_1^+ - u_1^-\right) - i\left(v_1^+ - v_1^-\right) = 0. \tag{4}$$

This additional condition aims to define the sought-for function $g(\theta)$ of displacements of the points of the external contour L of the hub.

3. Method of solution

We look for temperatures, stresses and displacements in the hub and plunger in the form of expansions in the small parameter where we ignore the terms containing ε of degree higher than the first. Each approximation satisfies the system of differential equations of plane thermoelasticity. To find the values of the temperature and the stress tensor components for $r = \rho(\theta)$ (similarly for $r = \rho_1(\theta)$), we expand in series the expressions for temperature, stress and displacements in the vicinity of $r = R_0$. Using the perturbations method, allowing for what has been said, we arrive at the sequence of boundary conditions for the problems of the plane theory of thermoelasticity for the hub.

For a zero approximation

$$A_{T1}\lambda\frac{\partial t^{(0)}}{\partial r} - A_{T2}\alpha_1^* t^{(0)} = -Q_*^{(0)}(\theta) \quad \text{for } r = R, \tag{5}$$

$$\begin{aligned}
&\lambda\frac{\partial t^{(0)}}{\partial r} + \alpha_2 t^{(0)} = 0 && \text{for } r = R_0,\\
&\sigma_r^{(0)} = -p^{(0)}(\theta), \tau_{r\theta}^{(0)} = -fp^{(0)}(\theta) && \text{on the contact area,}\\
&\sigma_r^{(0)} = 0, \tau_{r\theta}^{(0)} = 0 && \text{out of the contact area,}\\
&v_r^{(0)} - iv_\theta^{(0)} = g^{(0)}(\theta) && \text{for } r = R_0,\\
&\sigma_{y_1}^{(0)} = 0, \tau_{x_1y_1}^{(0)} = 0 && \text{on the crack faces;}
\end{aligned} \tag{6}$$

for a first approximation

$$\begin{aligned}
&A_{T1}\lambda\frac{\partial t^{(1)}}{\partial r} - A_{T2}\alpha_1^* t^{(1)} = -Q_*^{(1)}(\theta) && \text{for } r = R,\\
&\lambda\frac{\partial t^{(1)}}{\partial r} + \alpha_2 t^{(1)} = 0 && \text{for } r = R_0,
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\sigma_r^{(1)} = N - p^{(1)}(\theta), \tau_{r\theta}^{(1)} = T_t - fp^{(1)}(\theta) && \text{on the contact area for } r = R,\\
&\sigma_r^{(0)} = N,\ \tau_{r\theta}^{(0)} = T_t && \text{out the contact area,}\\
&v_r^{(1)} - iv_\theta^{(1)} = g^{(1)}(\theta) && \text{for } r = R_0,\\
&\sigma_{y_1}^{(1)} = 0, \tau_{x_1y_1}^{(1)} = 0 && \text{on the crack faces.}
\end{aligned} \tag{8}$$

Here

$$Q_*^{(1)}(\theta) = -Q_b^{(1)}(\theta) + \left[A_{T1}\lambda \frac{\partial^2 t^{(0)}}{\partial r^2} - A_{T2}\alpha_1^* \frac{\partial t^{(0)}}{\partial r}\right] H(\theta),$$
$$N = -H(\theta)\frac{\partial \sigma_r^{(0)}}{\partial r} + 2\tau_{r\theta}^{(0)} \frac{1}{R}\frac{dH(\theta)}{d\theta} \quad \text{for r} = \text{R}, \tag{9}$$
$$T_t = \left(\sigma_\theta^{(0)} - \sigma_r^{(0)}\right)\frac{1}{R}\frac{dH(\theta)}{d\theta} - H(\theta)\frac{\partial \tau_{r\theta}^{(0)}}{\partial r},$$

$t = T - T_c$ is the excess temperature for the hub.

In a similar way we can write boundary conditions at each approximation for a plunger.

Additional equations (4) accept the following form in a zero approximation

$$\left(u_1^{0+}(x_1, 0) - u_1^{0-}(x_1, 0)\right) - i\left(v_1^{0+}(x_1, 0) - v_1^{0-}(x_1, 0)\right) = 0; \tag{10}$$

in a first approximation

$$\left(u_1^{1+}(x_1, 0) - u_1^{1-}(x_1, 0)\right) - i\left(v_1^{1+}(x_1, 0) - v_1^{1-}(x_1, 0)\right) = 0. \tag{11}$$

Now let us construct the solution of the problem in a zero approximation.

The solution of the boundary value problems in the theory of heat conduction at each approximation is sought by the method of separation of variables. We find the distribution of the excess temperature $t = t^{(0)} + \varepsilon t^{(1)}$ for the hub in the following form

$$t^{(0)} = C_{10} + C_{20}\ln r + \sum_{k=1}^{\infty}\left(C_{10}^{(k)} r^k + C_{20}^{(k)} r^{-k}\right)\cos k\theta + \sum_{k=1}^{\infty}\left(A_{10}^{(k)} r^k + A_{20}^{(k)} r^{-k}\right)\sin k\theta,$$
$$t^{(1)} = C_{11} + C_{21}\ln r + \sum_{k=1}^{\infty}\left(C_{11}^{(k)} r^k + C_{21}^{(k)} r^{-k}\right)\cos k\theta + \sum_{k=1}^{\infty}\left(A_{11}^{(k)} r^k + A_{21}^{(k)} r^{-k}\right)\sin k\theta.$$

The constants C_{10}, C_{20}, $C_{10}^{(k)}$, $C_{20}^{(k)}$, $A_{10}^{(k)}$, $A_{20}^{(k)}$ are determined from boundary conditions (5) of the heat conduction theory problem in a zero approximation. The coefficient C_{11}, C_{21}, $C_{11}^{(k)}$, $C_{21}^{(k)}$, $A_{11}^{(k)}$, $A_{21}^{(k)}$ are found from boundary conditions (7) of the heat conduction theory problem in a first approximation. Because of their bulky form, we do not cite the appropriate formulas. To solve the thermoelasticity problem, at each approximation we use the thermoelastic potential of displacements [33]. In the problem under consideration, the thermoelastic potential of displacements F for the hub in zero and first approximations is determined by the solution of the differential equations

$$\Delta F^{(0)} = \beta t^{(0)}, \Delta F^{(1)} = \beta t^{(1)}, \beta = \frac{1+\mu}{1-\mu}\alpha. \tag{12}$$

Here α is the coefficient of linear temperature extension.

We look for the solution of Equation (12) in the form

$$F^{(0)} = \sum_{n=0}^{\infty}\left(f_n^0(r)\cos n\theta + f_n^{0*}(r)\sin n\theta\right),$$
$$F^{(1)} = \sum_{n=0}^{\infty}\left(f_n^{(1)}(r)\cos n\theta + f_n^{(1)*}(r)\sin n\theta\right).$$

For the functions $f_n^0(r), f_n^{0*}(r)$ we get the ordinary differential equations

$$\frac{d^2 f_n^0}{dr^2} + \frac{1}{2}\frac{df_n^0}{dr} - \frac{n^2}{r^2} f_n^0 = \beta F_n^0,$$
$$\frac{d^2 f_n^{0*}}{dr^2} + \frac{1}{2}\frac{df_n^{0*}}{dr} - \frac{n^2}{r^2} f_n^{0*} = \beta F_n^{0*}.$$

Particular solutions of differential equations are sought by the method of variation of constants

$$f_n^0 = \beta\left[-\ln r\int_{R_0}^{r}\rho F_0^0(\rho)d\rho + \int_{r}^{R}\rho F_0^0(\rho)\ln\rho d\rho\right],$$
$$f_n^0 = -\frac{\beta}{2n}\left[r^n\int_{r}^{R}F_n^0(\rho)\rho^{1-n}d\rho + r^{-n}\int_{R_0}^{r}\rho F_n^0(\rho)\rho^{1+n}d\rho\right],$$
$$f_n^{0*} = -\frac{\beta}{2n}\left[r^n\int_{r}^{R}F_n^{0*}(\rho)\rho^{1-n}d\rho + r^{-n}\int_{R_0}^{r}\rho F_n^{0*}(\rho)\rho^{1+n}d\rho\right].$$

After defining the thermoelastic potential of displacements in a zero approximation for the hub, by the known formulae [33] we calculate the approximate thermoelastic potential of the stress $\bar{\sigma}_r^{(0)}$, $\bar{\sigma}_\theta^{(0)}$, $\bar{\tau}_{r\theta}^{(0)}$ and displacements $\bar{\upsilon}_r^{(0)}$, $\bar{\upsilon}_\theta^{(0)}$ in the hub

$$\bar{\sigma}_r^{(0)} = -2G\left\{\left\{\frac{1}{r}\sum_{n=0}^{\infty}\left[\frac{\partial f_n^0}{\partial r}\cos n\theta + \frac{\partial f_n^{0*}}{\partial r}\sin n\theta\right] + \right.\right.$$
$$\left. + \frac{1}{r^2}\sum_{n=0}^{\infty}\left(-n^2\right)\left[f_n^0\cos n\theta + f_n^{0*}\sin n\theta\right]\right\},$$

$$\bar{\sigma}_\theta^{(0)} = -2G\sum_{n=0}^{\infty}\left[\frac{\partial^2 f_n^0}{\partial r^2}\cos n\theta + \frac{\partial^2 f_n^{0*}}{\partial r^2}\sin n\theta\right],$$
$$\bar{\tau}_{r\theta}^{(0)} = 2G\left\{\left(-\frac{1}{r^2}\right)\sum_{n=0}^{\infty}n\left[f_n^{0*}\cos n\theta - f_n^0\sin n\theta\right] + \right.$$
$$\left. + \frac{1}{r}\sum_{n=0}^{\infty}n\left[\frac{\partial f_n^{0*}}{\partial r}\cos n\theta - \frac{\partial f_n^0}{\partial r}\sin n\theta\right]\right\}$$

$$\bar{\upsilon}_r^{(0)} = \sum_{n=0}^{\infty}\left[\frac{\partial f_n^0}{\partial r}\cos n\theta + \frac{\partial f_n^{0*}}{\partial r}\sin n\theta\right],$$
$$\bar{\upsilon}_\theta^{(0)} = \frac{1}{r}\sum_{n=0}^{\infty}\left[(-n)\left(f_n^0\sin n\theta - f_n^{0*}\cos n\theta\right)\right].$$

The found stress and displacements for the hub will not satisfy boundary conditions (6). It is necessary to find for the hub the second stress–strain state $\bar{\bar{\sigma}}_r^{(0)}$, $\bar{\bar{\sigma}}_\theta^{(0)}$, $\bar{\bar{\tau}}_{r\theta}^{(0)}$, $\bar{\bar{\upsilon}}_r^{(0)}$, $\bar{\bar{\upsilon}}_\theta^{(0)}$ such that boundary conditions (6) are fulfilled.

Consequently, to determine the second stress–strain state, we have the boundary conditions

$$\bar{\bar{\sigma}}_r^{(0)} = -p^{(0)}(\theta) - \bar{\sigma}_r^{(0)}, \tag{13}$$

$$\begin{array}{ll} \bar{\bar{\tau}}_{r\theta}^{(0)} = -fp^{(0)}(\theta) - \bar{\tau}_{r\theta}^{(0)} & \text{on the contact area for } r=R, \\ \bar{\bar{\sigma}}_{r}^{(0)} = -\bar{\sigma}_{r}^{(0)}, \bar{\bar{\tau}}_{r\theta}^{(0)} = -\bar{\tau}_{r\theta}^{(0)} & \text{out of the contact area,} \\ \bar{\bar{v}}_{r}^{(0)} - i\bar{\bar{v}}_{\theta}^{(0)} = g^{(0)}(\theta) - \left(\bar{v}_{r}^{(0)} - i\bar{v}_{\theta}^{(0)}\right) & \text{for } r=R_0, \\ \bar{\bar{\sigma}}_{y_1}^{(0)} = -\bar{\sigma}_{y_1}^{(0)}, \bar{\bar{\tau}}_{x_1y_1}^{(0)} = -\bar{\tau}_{x_1y_1}^{(0)} & \text{on the crack faces.} \end{array} \tag{14}$$

By the Kolosov–Muskhelishvili formulas [32], we can write the boundary conditions of the problem in (13) and (14) in the form of a boundary value problem for finding the complex potential $\Phi^{(0)}(z)$, $\Psi^{(0)}(z)$ for the hub.

We look for the complex potentials in the form

$$\Phi^{(0)}(z) = \Phi_1^{(0)}(z) + \Phi_2^{(0)}(z), \Psi^{(0)}(z) = \Psi_1^{(0)}(z) + \Psi_2^{(0)}(z), \tag{15}$$

$$\Phi_1^{(0)}(z) = \sum_{k=-\infty}^{\infty} a_k z^k, \ \Psi_1^{(0)}(z) = \sum_{k=-\infty}^{\infty} b_k z^k,$$

$$\Phi_2^{(0)}(z) = \frac{1}{2\pi}\int_{-\ell_1}^{\ell_1} \frac{g_1^0(t)dt}{t - z_1},$$

$$\Psi_2^{(0)}(z) = \frac{1}{2\pi} e^{-2i\alpha_1} \int_{-\ell_1}^{\ell_1} \left[\frac{\overline{g_1^0(t)}}{t - z_1} - \frac{\bar{T}_1 e^{i\alpha_1}}{(t - z_1)^2} g_1^0(t)\right] dt.$$

Here $T_1 = te^{i\alpha_1} + z_1^0$; $z_1 = e^{-i\alpha_1}\left(z - z_1^0\right)$; $g_1^0(t)$ is the sought-for function, characterizing the jump of displacements at a zero approximation when passing through the crack line

$$g_1^0(x_1) = \frac{2G}{i(1+\kappa)}\frac{\partial}{\partial x_1}\left[u_1^{0+}(x_1,0) - u_1^{0-}(x_1,0) + i\left(v_1^{0+}(x_1,0) - v_1^{0-}(x_1,0)\right)\right], \tag{16}$$

$$\kappa = 3 - 4\mu.$$

We represent the boundary value problem for finding the complex potential on circular boundaries in the form

$$\Phi_1^{(0)}(\tau_0) + \overline{\Phi_1^{(0)}(\tau_0)} - e^{2i\theta}\left[\bar{\tau}_0 \Phi_1'^{(0)}(\tau_0) + \Psi_1^{(0)}(\tau_0)\right] = X^{(0)}(\theta) - (f_1 - if_2), \tag{17}$$

$$\begin{array}{l} \Phi_1^{(0)}(\tau) - \kappa\overline{\Phi_1^{(0)}(\tau)} - e^{2i\theta}\left[\bar{\tau}\Phi_1'^{(0)}(\tau) + \Psi_1^{(0)}(\tau)\right] = g^{(0)}(\theta) - (f_3 - if_4), \\ \tau = R_0 \exp(i\theta), \tau_0 = R \exp(i\theta), \end{array},$$

$$X^{(0)}(\theta) = \begin{cases} -(1 - if)p^{(0)}(\theta) - \left(\bar{\sigma}_r^{(0)} - i\bar{\tau}_{r\theta}^{(0)}\right) & \text{on the contact area} \\ -\left(\bar{\sigma}_r^{(0)} - i\bar{\tau}_{r\theta}^{(0)}\right) & \text{out of the contact area} \end{cases}$$

$$f_1 - if_2 = \Phi_2^{(0)}(\tau_0) + \overline{\Phi_2^{(0)}(\tau_0)} - e^{2i\theta}\left[\bar{\tau}_0 \Phi_2^{(0)}(\tau_0) + \Psi_2^{(0)}(\tau_0)\right],$$

$$f_3 - if_4 = \Phi_1^{(0)}(\tau) - \kappa\overline{\Phi_1^{(0)}(\tau)} - e^{2i\theta}\left[\bar{\tau}\Phi_1'^{(0)}(\tau) + \Psi_1^{(0)}(\tau)\right] - \left(\bar{v}_r^{(0)} - i\bar{v}_\theta^{(0)}\right).$$

To solve the boundary value problem (17) with respect to potentials $\Phi_1^{(0)}(z)$ and $\Psi_1^{(0)}(z)$, we use the method of power series. For this, the right-hand sides of conditions (16) are expanded into Fourier series

$$X^{(0)}(\theta) = \sum_{k=-\infty}^{\infty} A_k^{(0)} e^{ik\theta},$$

$$-(f_1 - if_2) = \sum_{k=-\infty}^{\infty} D_k^{(0)} e^{ik\theta}, \; -(f_3 - if_4) = \sum_{k=-\infty}^{\infty} F_k^{(0)} e^{ik\theta}.$$

The coefficients $D_k^{(0)}$ and $F_k^{(0)}$ are expressed in the form of integrals from the sought-for function $g_1^{(0)}$. To determine them, the residue method was used. After some transformations we arrive at the infinite linear algebraic system with respect to the coefficients a_k and b_k, whose solution is written in the form

$$a_0 = \frac{\left(A_0^{(0)} + D_0^{(0)}\right)R_0^2 - F_0^{(0)}R^2}{2R_0^2 - (1-\kappa)R^2}, a_{-1} = \frac{\left(A_1^{(0)} + D_1^{(0)}\right)R_0}{1+\kappa},$$

$$b_{-2}R_0^{-2} = 2a_0 - A_0^{(0)} - D_0^{(0)}, b_{-1} = \frac{\kappa\left(\overline{A_1^{(0)}} + \overline{D_1^{(0)}}\right)R_0}{1+\kappa},$$

$$a_k = \frac{(1+\kappa)\left(R^2 - R_0^2\right)M_k - \bar{M}_{-k}\left(R_0^{-2k+2} + \kappa R^{-2k+2}\right)}{(1-\kappa^2)\left(R^2 - R_0^2\right)^2 - \left(R_0^{-2k+2} + \kappa R^{-2k+2}\right)\left(R_0^{2k+2} + \kappa R^{2k+2}\right)} \; (k = \pm 2, \pm 3, \ldots),$$

$$M_k = F_k^{(0)} R^{-k+2} - \left(A_k^{(0)} + D_k^{(0)}\right) R_0^{-k+2},$$

$$a_1 = \frac{2\left(A_1^{(0)} + D_1^{(0)}\right)\left(R^2 - R_0^2\right)R_0}{(1-\kappa)\left(R_0^4 + \kappa R^4\right)} - \frac{\bar{M}_{-1}}{R_0^4 + \kappa R^4},$$

$$b_{k-2}R_0^{k-2} = (1-\kappa)a_k R_0^k + \bar{a}_k R_0^{-k} - \left(A_k^{(0)} + D_k^{(0)}\right).$$

The right-hand sides of these formulas contain the coefficients of expansion of the function $g^{(0)}(\theta)$ of displacements of the points of the external contour L of the hub in a zero approximation

$$g'^{(0)}(\tau) = \sum_{k=-\infty}^{\infty} A_k^0 e^{ik\theta} = a_0^{*(0)} + \sum_{k=1}^{\infty} \left(a_k^{*(0)} \cos k\theta + b_k^{*(0)} \sin k\theta\right)$$

and the coefficients of expansion of the contact pressure

$$p^{(0)}(\theta) = \alpha_0^0 + \sum_{k=1}^{\infty} \left(\alpha_k^0 \cos k\theta + \beta_k^0 \sin k\theta\right)$$

and also the integrals of the sought-for function $g_1^{(0)}(t)$.

Satisfying by functions (15) the boundary conditions on the crack faces (14), we get a singular integral equation with respect to the unknown function $g_1^{(0)}(x_1)$

$$\int_{-\ell_1}^{\ell_1} \left[R(t,x_1)\, g_1^0(t) + S(t,x_1)\, \overline{g_1^0(t)}\right] dt = \pi f_1(x_1), \tag{18}$$

$$f_1(x_1) = -\left[\Phi_1^{(0)}(x_1) + \overline{\Phi_1^{(0)}(x_1)} + x_1 \overline{\Phi_1'^{(0)}(x_1)} + \overline{\Psi_1^{(0)}(x_1)}\right] - \left(\bar{\sigma}_{y_1}^{(0)} - \bar{\tau}_{x_1 y_1}^{(0)}\right).$$

The variables x_1, t, ℓ_1 are dimensionless quantities referring to R_0; $R(t,x_1)$ and $S(t,x_1)$ are determined by the relations of formula VI.61 in Panasyuk et al. [34].

To the singular integral equation for the inner crack it is necessary to add the condition of uniqueness of displacements by passing the contour of the crack

$$\int_{-\ell_1}^{\ell_1} g_1^0(t)dt = 0. \tag{19}$$

By means of the algebraization process [34–36], the complex singular integral equation (18) of the above-remarked condition (19) is reduced to the system of M complex algebraic equations for defining M unknowns $g_1^0(t_m) = v_1^0(t_m) - iu_1^0(t_m)$ ($m = 1,2,\ldots,M$)

$$\frac{1}{M}\sum_{m=1}^{M} \ell_1 \left[g_1^0(t_m)R(\ell_1 t_m, \ell_1 x_r) + \overline{g_1^0(t_m)}\, S(\ell_1 t_m, \ell_1 x_r)\right] = f_1(x_r), \tag{20}$$

$$\sum_{m=1}^{M} g_1^0(t_m) = 0 \quad r = 1, 2, \ldots, M-1,$$

$$t_m = \cos\frac{2m-1}{2M}\pi, \quad x_r = \cos\frac{\pi r}{M}.$$

If in (20) we pass to complexly conjugated values, we get one more M algebraic equation. The right-hand side of system (20) contains unknown values of coefficients of expansion of the functions of displacements of the points of the outer contour L of the hub and contact pressure $p^{(0)}(\theta)$.

By means of complex potentials (15), Kolosov–Muskhelishvili formulas and integration of kinetic equation of wear (2) of the hub material, we find radial displacement v_1^0 of the contact surface of the hub at a zero approximation.

A thermoelasticity problem for a plunger is considered in the same way. By using the solution of the thermoelasticity problem for a plunger in a zero approximation and the kinetic equation of wear of the plunger material we find radial displacement v_2^0 of the contact surface of the plunger in a zero approximation. The found quantities v_1^0 and v_2^0 are substituted into the main contact equation (1) in a zero approximation.

For algebraization of the main contact equation, the unknown functions of contact pressure in zero approximation are sought in the form of expansions

$$\begin{aligned} p^{(0)}(\theta, t) &= p_0^0(\theta) + tp_1^0(\theta) + \ldots, \\ p_s^0(\theta) &= \alpha_0^s + \sum_{k=1}^{\infty}\left(\alpha_k^s \cos k\theta + \beta_k^s \sin k\theta\right) \quad (s = 0, 1, \ldots). \end{aligned} \tag{21}$$

Substituting relations (21) into the main contact equation in a zero approximation, we find functional equations for sequential determination of $p_0^0(\theta)$, $p_1^0(\theta)$, etc. For constructing an algebraic system with respect to α_k, β_k, we equate the coefficients at the identical trigonometric functions in the left- and right-hand sides of the functional equation of the contact problem. As a result, we get an infinite algebraic system with respect to α_k^0 ($k = 0,1,2,\ldots$), β_k^0 ($k = 1,2,\ldots$) and α_k^1, β_k^1, etc. Because of unknown quantities θ_1 and θ_2, the system of equations becomes nonlinear.

To determine the quantities θ_1 and θ_2 ($\theta_1 = \theta_1^0 + \varepsilon\theta_1^1 + \ldots$, $\theta_2 = \theta_2^0 + \varepsilon\theta_2^1 + \ldots$) we have condition (3). We can represent this equation in the form

$$p^{(0)}\left(\theta_1^0\right) = 0,\, p^{(0)}\left(\theta_2^0\right) = 0 \text{ for a zero approximation,} \tag{22}$$

$$p^{(1)}\left(\theta_1^1\right) = 0,\, p^{(1)}\left(\theta_2^1\right) = 0 \text{ for a first approximation.} \tag{23}$$

The right-hand sides of infinite algebraic systems with respect to α_k, β_k contain the integrals from the unknown function $g_1^0(x_k)$ and also unknown values of the coefficients of expansion of the function $g^0(\theta)$ of displacements of the point of the outer contour L of the hub in a zero approximation. Thus, infinite algebraic systems with respect to α_k, β_k and the finite system with respect to $g_1^0(x_k)$ are connected between themselves and they should be solved jointly. The obtained systems of equations with respect to a_k, b_k, α_k, β_k, $g_1^0(t_m)$ ($m = 1,2,\ldots,M$) at the given function of displacements of the points of the outer contour L of the hub attempt to find, in a zero approximation, the stress–strain state of the hub of a friction pair involving a crack in the hub, contact pressure, stress intensity factors in the vicinity of the crack tip, temperature distribution and also details of the abrasive wear of the friction pair.

In the stated optimal design problem, it is necessary to determine the function of displacements of the points of the outer contour L of the hub. The coefficients A_k^0 $(k = 0,\pm 1,\pm 2,\ldots)$ should be defined. Consequently, the obtained combined algebraic system is not still closed.

To construct missing equations in a zero approximation, we require the conditions (10) to be fulfilled at the nodal points belonging to the end areas, where the closure of the crack faces must happen. In the case under consideration, instead of (10) it is convenient to use the expression for the derivative of the crack faces opening. Thus, the missing equations in a zero approximation are obtained in the form

$$g_1^0(t_k) = 0 \quad (k = 1, 2, \ldots, M_1), \tag{24}$$

where M_1 is the number of nodal points belonging to the end areas.

4. A method for numerical solution

The joint solution of the obtained systems of equations attempts to find the approximate values of the coefficients a_k, b_k, α_k, β_k, the values of crack faces opening functions $v_1^0(t_m)$, $u_1^0(t_m)$ and the coefficients of the function of displacements of the points of the outer contour L of the hub A_k^0 $(k = 0,\pm 1,\pm 2,\ldots,\pm M_1)$. As we give beforehand the end area sizes, the system of equations (23) becomes linear. The combined system of algebraic equations will be nonlinear because of unknown quantities θ_1^0 and θ_2^0. For solving it we use the reduction and successive approximations methods, whose essence is the following: we use the combined system algebraic at some certain values of θ_1^{0*} and θ_2^{0*} with respect to the remaining unknowns (listed above). The remaining unknowns linearly enter the combined system. The values θ_1^{0*}, θ_2^{0*} and the remaining unknowns are substituted into the unused equations (22). The taken values θ_1^{0*}, θ_2^{0*} and the values of the remaining unknowns corresponding to them will not, generally speaking, satisfy Equations (22). Therefore, choosing the values of parameters θ_1^0 and θ_2^0, we will repeat the calculations until the last equations of system (22) will be satisfied with the given accuracy.

After defining the sought-for quantities of a zero approximation, we can construct the solution of the inverse problem in a first approximation. The functions N and T_t are determined on the basis of the obtained solution for R. Boundary conditions (8) are written in the form of a boundary value problem for finding the complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$. We look for the complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$ in a similar way as (14) with obvious changes. The further course of the solution is the same as in a zero approximation.

The obtained complex singular integral equation with respect to $g_1^1(t)$ under the additional condition of type (19) by means of the algebraization procedure is reduced to the finite system of M algebraic equations for defining M unknowns

$$g_1^1(t_m)(m = 1, 2, \ldots, M).$$

The right-hand sides of this system contain the coefficients of expansion of the function $g^{(1)}(\theta)$ of displacements of the points of the outer contour L of the hub in a first approximation

$$g'^{(1)}(\theta) = \sum_{k=-\infty}^{\infty} A_k^1 e^{ik\theta} = a_0^{*(1)} + \sum_{k=1}^{\infty} \left(a_k^{*(1)} \cos k\theta + b_k^{*(1)} \sin k\theta\right),$$

the contact pressure

$$p^{(1)}(\theta) = \alpha_0^1 + \sum_{k=1}^{\infty} \left(\alpha_k^1 \cos k\theta + \beta_k^1 \sin k\theta\right)$$

and also the integrals of the function $g_1^1(t_m)$.

Construction of the missing equations for defining the unknown contact stresses is realized similar to a zero approximation. A thermoelasticity problem for a hub in a first approximation is solved in the same way. Algebraization of the main equations of the contact problem in a first approximation is conducted just as in a zero approximation. The sought-for functions of contact pressure are represented in the form

$$p^{(1)}(\theta) = p_0^1(\theta) + tp_0^1(\theta) + \dots, \tag{25}$$

$$p_0^1(\theta) = \alpha_{0,0}^1 + \sum_{k=1}^{\infty} \left(\alpha_{k,0}^1 \cos k\theta + \beta_{k,0}^1 \sin k\theta\right),$$

$$p_1^1(\theta) = \alpha_{0,1}^1 + \sum_{k=1}^{\infty} \left(\alpha_{k,1}^1 \cos k\theta + \beta_{k,1}^1 \sin k\theta\right).$$

As a result, we get infinite linear algebraic systems with respect to $\alpha_{0,0}^1$, $\alpha_{k,0}^1$, $\beta_{k,0}^1$ ($k = 1,2,\dots$) and $\alpha_{0,1}^1$, $\alpha_{k,1}^1$, $\beta_{k,1}^1$ ($k = 1,2,\dots,n$).

Because of unknown quantities θ_1^1 and θ_2^1, the system of equations becomes nonlinear.

The obtained systems of equations with respect to a_k^1, b_k^1, α_k^1, β_k^1, $g_1^1(t_m)$ ($m = 1,2,\dots,M$) attempt, at the given function of displacements of the point of the outer contour L of the hub, to find in a first approximation the stress–strain state of the hub of a friction pair involving a crack in the hub, contact pressure, stress intensity factors in the vicinity of crack tips, temperature distribution and also abrasive wear of the hub and the plunger.

In the optimal design problem under consideration, the coefficients A_k^1 ($k = 0,\pm 1,\pm 2,\dots$) of the function of displacements of the point of the outer contour L of the hub should be defined. To construct the missing equations in a first approximation, we require the additional conditions (11) to be fulfilled at the nodal points belonging to the end areas at which the crack faces closure occurs. As in a zero approximation, instead of (11) it is convenient to use the relation for the derivative of the crack faces opening. Consequently, the missing equations in a first approximation are in the form

$$g_1^1(t_k) = 0 \quad (k = 1, 2, \dots, M_1). \tag{26}$$

5. Numerical results and their analysis

Because of the unknown ends of the contact area (quantities θ_1 and θ_2), the combined system of equations becomes nonlinear. The constructed combined system of equations is closed and attempts at the given functions $H(\theta)$ and $H_1(\theta)$ to find by the numerical calculations the optimal function of displacements of the points of the outer contour L of the hub, contact pressure, stress–strain state, temperature and wear of the hub and plunger of a friction pair.

The functions $H(\theta)$ and $H_1(\theta)$ describing the roughnesses of the inner surface of the hub and the plunger were considered as a determined set of the rough surfaces of the contours profiles and were modeled by a stationary random function with a zero mean value and known variance. The problem under consideration has many free parameters. These are various thermophysical and mechanical characteristics of the materials, geometrical sizes of the hub and the velocity of the plunger motion. The results of the calculations of contact pressure for the hub of a slush pump depending on the value of the polar angle $\theta' = \theta - \theta_+ \left(\theta_0 = \frac{\theta_2 - \theta_1}{2};\ \theta + = \frac{\theta_2 + \theta_1}{2}\right)$ are represented in the form of graphs in Figure 2 at velocity of the plunger $V = 0.2$ m/s for the case of a rectilinear crack for $\alpha_1 = 60^0$; curve 1 corresponds to the optimal solution and curve 2 corresponds to the case when the function of the displacements of the external contour points of the hub of a friction pair is equal to zero. The following were accepted as parameters: $2R = 57$ mm; $2R_0 = 73$ mm; $2R' = 56.7$mm; $f = 0.2$; $E = 1.8 \cdot 10^5$ MPa; $E_1 = 2.1 \cdot 10^5$ MPa; $\nu = 0.25$; $\nu_1 = 0.3$; $K^{(1)} = 2 \cdot 10^{-8}$; $K^{(2)} = 2.5 \cdot 10^{-9}$; $\Delta_1 = R - R' = 0.15$ mm.

The maximum values of contact pressure, as a rule, are found in the middle part of the contact surface subject to the wrapping angle and friction coefficient. The existence of friction forces in the contact area reduces to the shift of the graph of contact pressure distribution in the direction opposite to the action of the moment. The results of calculations of the function of displacements of the points of the outer contour L of the hub are given in Table 1 (the coefficients are given in mm).

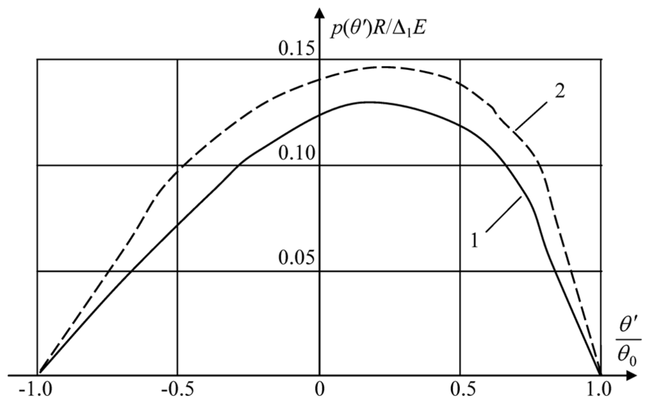

Figure 2. Distribution of contact pressure for the hub of a slush pump depending on the value of the polar angle.

Table 1. Values of Fourier coefficients of functions $g(\theta)$ describing displacements for hub external contour points for plunger speed $V = 0.2$ m/s.

a_0^*	a_1^*	a_2^*	a_3^*	a_4^*	a_5^*	a_6^*	a_7^*	a_8^*	a_9^*
0.1103	0.0893	0.7250	0.0632	0.0566	0.0498	0.0407	0.0322	0.0289	0.0208
	b_1^*	b_2^*	b_3^*	b_4^*	b_5^*	b_6^*	b_7^*	b_8^*	b_9^*
	0.0813	0.0709	0.0579	0.0508	0.0426	0.0358	0.0271	0.0205	0.0143

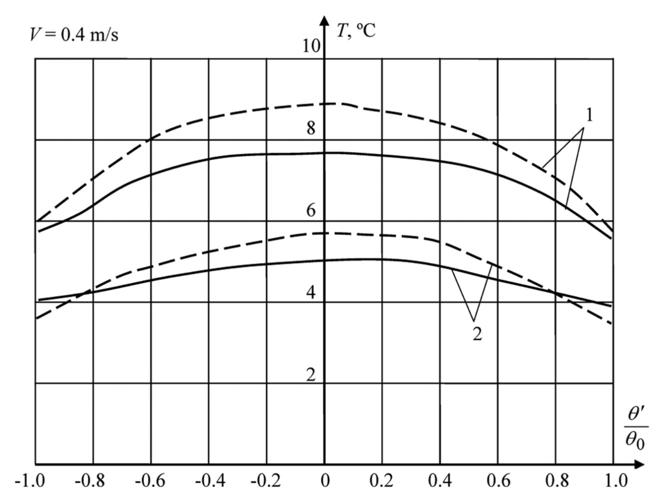

Figure 3. Temperature change in the hub for one stroke of the plunger for velocity of plunger motion $V = 0.4$ m/s.

With known contact pressure it is possible to calculate the temperature distribution and abrasive wear of the friction pair. We studied the temperature change for different depths on the thickness of the hub. Graphs of the temperature change in the hub for one stroke of the plunger for different velocities of plunger motion are represented in Figures 3 and 4. Here solid lines correspond to the optimal solution; dashed lines correspond to case when the function of displacements of the points of the external contour of the hub is equal to zero. Curve 1 corresponds to the surface temperature of the hub and curve 2 corresponds to temperature at the depth of 1.5 mm.

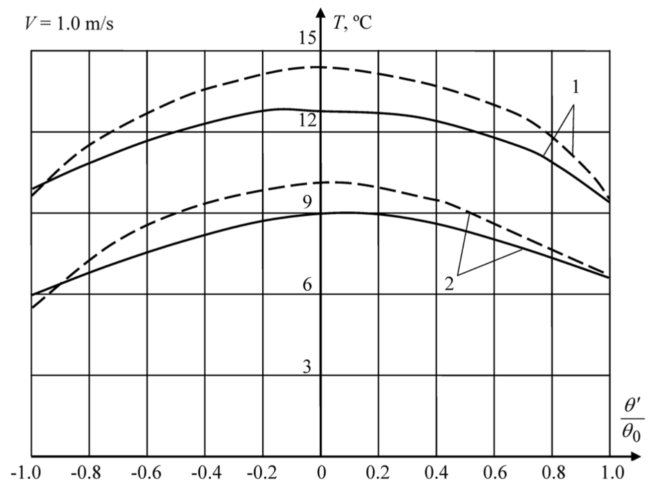

Figure 4. Temperature change in the hub for one stroke of the plunger for velocity of plunger motion V = 1.0 m/s.

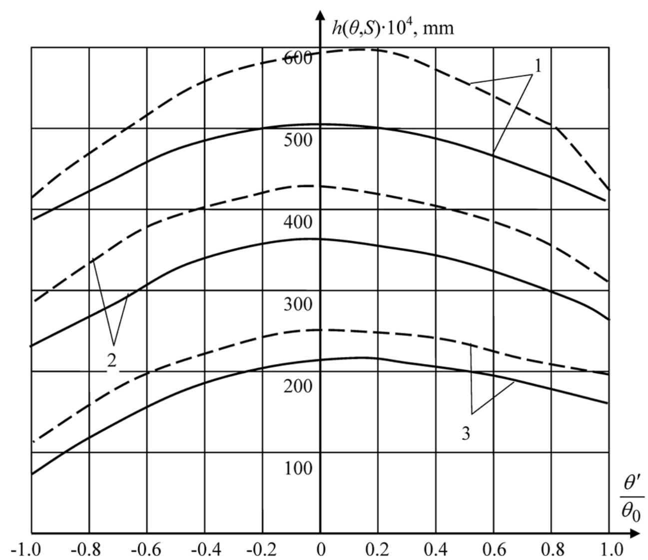

Figure 5. Abrasive wear of the hub surface for one stroke of the plunger.

The results of calculations of the abrasive wear of the hub surface for one stroke of the plunger are shown in Figure 5 for different velocities of the plunger motion. Here curves 1–3 correspond to the velocity $V = 1.0, 0.5, 0.2$ m/s, respectively.

Analysis of the calculation results shows that, at low values of the contact pressure, the hub wear along the length of the contact zone has an uneven character and is mainly formed by the change of the contact pressure. With increasing of the contact pressure, the hub wear along the length of the contact zone tends to leveling, and mainly depends on the wear (friction) path. The friction coefficient of the pair has a significant effect on the wear of the friction pair hub. In this connection, the dependence of maximum wear on the friction coefficient was calculated. In Figure 6 the dependence of maximum wear is shown for the friction path corresponding to 10 work hours of the pump. Here curve 1 corresponds to the optimal solution and curve 2 corresponds to the case when the function of displacements of the external contour points of the hub is equal to zero.

In the course of time, the surface microgeometry of the hub and plunger will vary due to the wear. The work relations obtained for the radial wear allow one to determine the variation of friction surface on considering the moment of time.

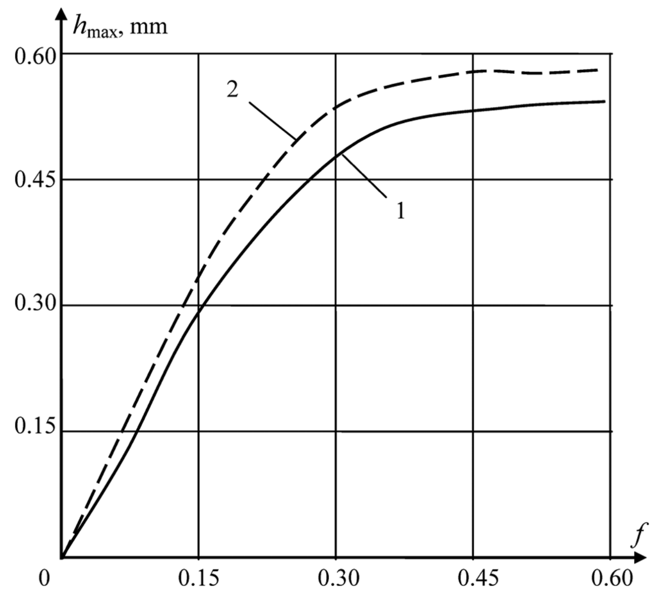

Figure 6. Dependence of maximum wear for the friction path corresponding to 10 work hours of the pump.

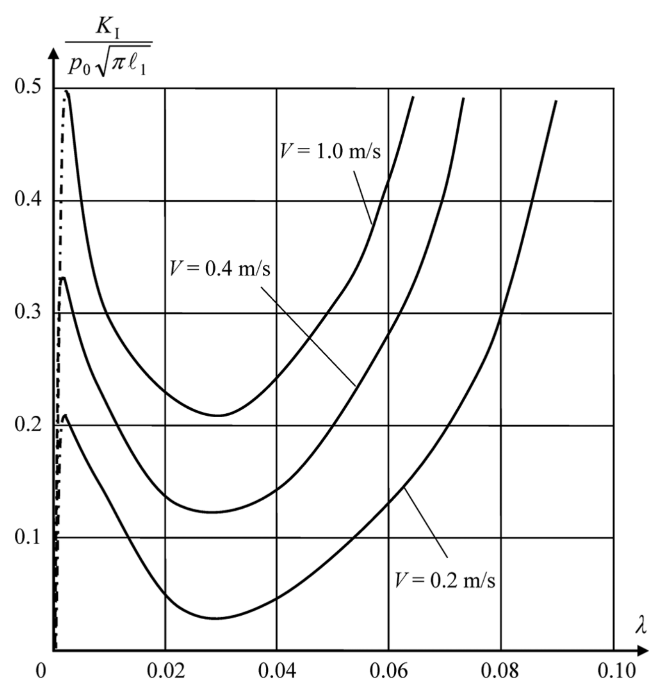

Figure 7. Dependence of the stress intensity factor K_I on the parameter $\lambda = \ell_1/(R_0 - R)$.

Dependence of the stress intensity factors on the parameter $\lambda = \ell_1/(R_0 - R)$ is shown in Figures 7 and 8 for the different velocities of the plunger motion for the function of the displacements of the external contour points of the hub of a friction pair equal to zero. Here, $p_0 = \frac{P}{4\pi E\Delta_1}$. In the calculations it was assumed that $M = 20; 40$. Numerous studies show that the number of collocations M should not be less than 20. The infinite algebraic systems, respectively, the coefficients α_k, β_k, were reduced to $k = 5; 7; 9$ equations. The system (19) that serving to find the Fourier coefficients of the sought-for function also

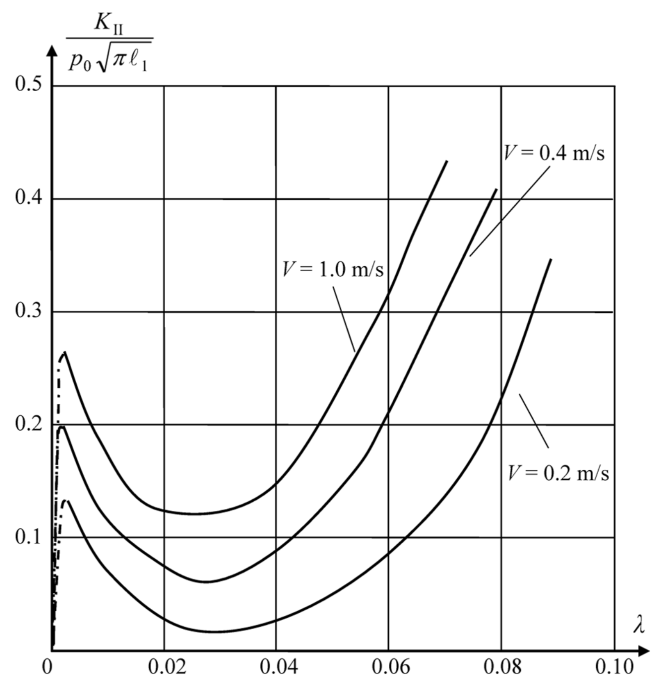

Figure 8. Dependence of the stress intensity factor K_{II} on the parameter $\lambda = \ell_1/(R_0 - R)$.

was reduced to $m_1 = 5$; 7; 9 equations. It should be noted that the values of the sought-for coefficients are not substantially changed, since $M = 20$, $k = 5$, $m_1 = 5$.

Changing the values of the parameters z_1^0 and α_1 characterizing the state of the crack, we can investigate different cases of the location of a crack in a hub. If a crack with one end reaches the inner surface of the hub, then quality (19) is replaced by a condition expressing the finiteness of stresses at the crack end. At the place where the crack faces interact, that is, the end areas, there arise normal $q_{y_1}(x_1)$ and tangential $q_{x_1y_1}(x_1)$ contact stresses. To determine them, at the already known function of displacements $g(\theta)$ of the points of the outer contour L of the hub, it is necessary again to solve a fracture mechanics problem for a crack with partially contacting faces. A method for solving such problems was given by Mirsalimov and Mustafayev [37].

6. Conclusions

The practice of the operation of friction pairs shows that at the stage of design of new constructions of mobile conjugations, it is necessary to take into account the cases when in separate friction nodes (hub) there may arise cracks. The main resolving equations found in the paper attempt, at the given function of displacements of the points of the outer contour of the hub, by numerical calculations to find the stress intensity factors to predict the growth of the existing crack in the hub, to establish the admissible size of a defect and to find the maximum values of workloads providing a sufficient safety margin. The solution of the inverse problem to the definition of the function of displacements of the points of the outer contour of the hub enables, at the design stage, to establish the optimal geometrical parameters of the elements of mobile conjugations, providing an increase of the load-bearing capacity of the friction pair.

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References

[1] Goryacheva, IG. *Contact mechanics in tribology*. Dordrecht: Kluwer Acad. Publ., 1998.
[2] Goryacheva, IG. *Mechanics of frictional interaction*. Moscow: Nauka, 2001 (in Russian).
[3] Thomas, TR. *Rough surface*. London: Longman, 1982.
[4] Bhushan, B. Contact mechanics of rough surfaces in tribology: multiple asperity contact. *Tribol Lett* 1998; 4: 1–35.
[5] Carbone, G, and Bottiglione, F. Contact mechanics of rough surfaces: a comparison between theories. *Meccanica* 2011; 46: 557–565.
[6] Czifra, A, and Horvath, S. Complex microtopography analysis in sliding friction of steel-ferodo material pair. *Meccanica* 2011; 46: 609–616.
[7] Gadzhiev, GH, and Mirsalimov, VM. Minimizing wear of the internal surface of a split cylinder sleeve in a contact pair. *J Friction Wear* 2004; 25: 231–237.
[8] Gadzhiev, GH, and Mirsalimov, VM. Optimal design of the composite cylinder–piston contacting pair. *J Friction Wear* 2004; 25: 466–473.
[9] Peigney, M. Simulating wear under cyclic loading by a minimization approach. *Int J Solids Struct* 2004; 41: 6783–6799.
[10] Mirsalimov, VM. Minimization of stress state of contact pair bushing. *J Friction Wear* 2006; 27: 388–393.
[11] Mirsalimov, VM. Minimization of thermal state of contact pair bushing. *J Mach Manuf Reliab* 2006; 35: 88–95.
[12] Chu, HM. Shape optimum design of slider bearings using inverse method. *Tribol Int* 2007; 40: 906–914.
[13] Mirsalimov, VM. An inverse wear contact problem for a friction couple. *J Mach Manuf Reliab* 2008; 37: 53–59.
[14] Mirsalimov, VM. Inverse problem of fracture mechanics for a disk fitted onto a rotating shaft. *J Appl Mech Tech Phys* 2009; 50: 712–719.
[15] Chu, LM, Chang, YP, and Yang, JH. Profile design of piston ring using inverse method. *J Marine Sci Technol* 2008; 16: 64–70.
[16] Alyaqout, SF, and Elsharkawy, AA. Optimum shape design for surface of a porous slider bearing lubricated with couple stress fluid. *Lubricat Sci* 2009; 21: 1–12.
[17] Mirsalimov, VM. Minimizing contact pressure of the bush-plunger friction pair. *J Friction Wear* 2009; 30: 25–32.
[18] Xu, G, Zhang, J, Wang, J, et al. Optimum design with numerical algorithm of hyperboloidal sliding bearing based on the reliability constraints. *Adv Sci Lett* 2011; 4: 2463–2467.
[19] Huang, GQ, Yu, J, and Pu, HL. Optimal design on clearance of piston friction pair for water hydraulic pump. *Adv Mater Res* 2011; 287-290: 3056–3060.
[20] Hussain, FS, and Sonpimple, MK. Application of non-traditional method for the optimization of friction clutch. *J Theor Appl Inform Technol* 2013; 49: 481–487.
[21] Mirsalimov, VM, and Veliyev, FE. Inverse problem of failure mechanics for a drawing die strengthened with a holder. *Acta Polytechnica Hungarica* 2013; 10: 121–138.
[22] Paredes, M, Canivenc, R, and Sartor, M. Tolerance optimization by modification of Taguchi's robust design approach and considering performance levels: application to the design of a cold-expanded bushing. *Proc Inst Mech Eng G J Aerospace Eng* 2014; 228: 1314–1323.
[23] Mirsalimov, VM, and Akhundova, PE. Minimization of contact pressure for hub–shaft friction pair. *J Friction Wear* 2015; 36: 404–408.
[24] Chernets, M. Evaluation of the contact strength and durability of plain bearings with different types of shaft lobing. *Proc Inst Mech Eng J J Eng Tribol* 2015; 229: 1444–1454.
[25] Huang, R, Li, T, Yu, X, et al. An optimization of the star-wheel profile in a single screw compressor. *Proc Inst Mech Eng A J Power Energ* 2015; 229: 139–150.
[26] Abdo, J. Materials sliding wear model based on energy dissipation. *Mech Adv Mater Struct* 2015; 22: 298–304.
[27] Zhang, Z, Liu, J, and Xie, Y. Design approach for optimization of a piston ring profile considering mixed lubrication. *Friction* 2016; 4: 335–346.
[28] Mirsalimov, VM, and Akhundova, PE. Minimization of abrasive wear for the internal surface of the hub of a friction pair. *J Friction Wear* 2016; 37: 424–429.
[29] Zhang, Z, Liu, J, Tang, Y, et al. Optimizing the shape of top piston ring face using inverse method. *Ind Lubricat Tribol* 2016; 68: 9–15.
[30] Mirsalimov, VM, and Akhundova, PE. Optimal design of a frictional pair of a hub–plunger. *J Friction Wear* 2017; 38: 384–389.
[31] Mirsalimov, VM, and Akhundova, PE. Inverse problems of damage mechanics for a hub of a friction pair. *Int J Damage Mech* 2018; 27: 82–96.
[32] Muskhelishvili, NI. *Some basic problem of mathematical theory of elasticity*. Amsterdam: Kluwer, 1977.
[33] Parkus, H. *Instationäre wärmespannungen*. Wien: Springer, 1959.
[34] Panasyuk, VV, Savruk, MP, and Datsyshyn, AP. *The stress distribution around cracks in plates and shells*. Kiev: Naukova dumka, 1976 (in Russian).
[35] Mirsalimov, VM. *Non-one-dimensional elastoplastic problems*. Moscow: Nauka, 1987 (in Russian).

[36] Ladopoulos, EG. *Singular integral equations: linear and non-linear theory and its applications in science and engineering*. Berlin: Springer, 2000.
[37] Mirsalimov, VM, and Mustafayev, AB. Inhibition of a curvilinear bridged crack by induced thermoelastic stress field. *J Therm Stress* 2016; 39: 1301–1319.