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Minimization of the Thermal State of the Hub of a Frictional Pair Using the Criterion of Uniform Temperature Distribution on a Friction Surface

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Abstract—Based on the harmonic model of a rough friction surface, the theoretical analysis of the determination of the function of the displacement of the points of a hub of the external surface and microgeometry of the friction surface ensuring the uniform temperature distribution on the contact surface is conducted. The closed set of algebraic equations is constructed, which makes it possible to solve the problem of the optimal design of a frictional pair depending on the geometric and mechanical characteristics of its elements.

Keywords: frictional pair, hub, plunger, uniform temperature distribution

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INTRODUCTION

The effectiveness of the operation of many types of frictional pairs largely depends on temperature. During a long period of operation the plunger with a large force rubs against the hub surface. The surface layer of the hub's metal is intensively heated. Because of the large heat conductivity, the heated surface layer of the hub is very quickly cooled and is hardened. Such a sharp hardening of a thin layer promotes the appearance of cracks on the hub. It is shown in [1–3] that at the spots of that actually touch a powerful heating occurs in the thin surface layers. This causes the appearance of regions of crack formation. In the case of powerful heating under the action of a temperature spike, the cracks originate in the near-surface layer. The destruction is caused by the heat production during friction. Thus, it can be assumed that each material has a limit (allowable) temperature T_*, exceeding which leads to the formation of burns and regions of microcracks in the material of frictional pairs. Consequently, the magnitude of the maximum temperature reached in the material can be considered the cause of the thermal destruction of the materials of the frictional pair.

Thermal failure in the development of friction nodes can be controlled by design and technological methods, in particular the geometry of the tribological conjunction, at the design stage. Solutions of the mechanics of problems by constructing such a geometry of the elements of the surface of the frictional pair, which would promote the reduction of the thermal stress and wear, with the exception of [4–6], are not known. The solution of this problem will make it possible to increase the working capacity of tribological conjunctions.

WORK OBJECTIVE

The work objective is the development of a mathematical model for the frictional interaction and wear for a hub-plunger pair, making it possible to calculate the optimal function of displacements of the points of the external contour of the hub and the microgeometry of the friction surface, when the temperature distribution on the contact surface is close to uniform during the specified operating modes of the pair.

PROBLEM STATEMENT

Let us conduct a theoretical analysis regarding the definition of the function of displacements of the points of the external hub surface and microgeometry of the friction surface, which would ensure that the temperature distribution of the contact surface is close to uniform. Thus, by choosing the displacements of the external hub surface and microgeometry of the friction surface, we can reduce the level (concentration) of the temperature. We use the least squares principle. We take the parameters of the function of the displacements of the points of the external hub contour and microgeometry of the friction surface as the control variables. We use the differential equations of thermoelasticity with the appropriate boundary conditions, which the components of the stresses and displacement vector in the hub and plunger should sat-

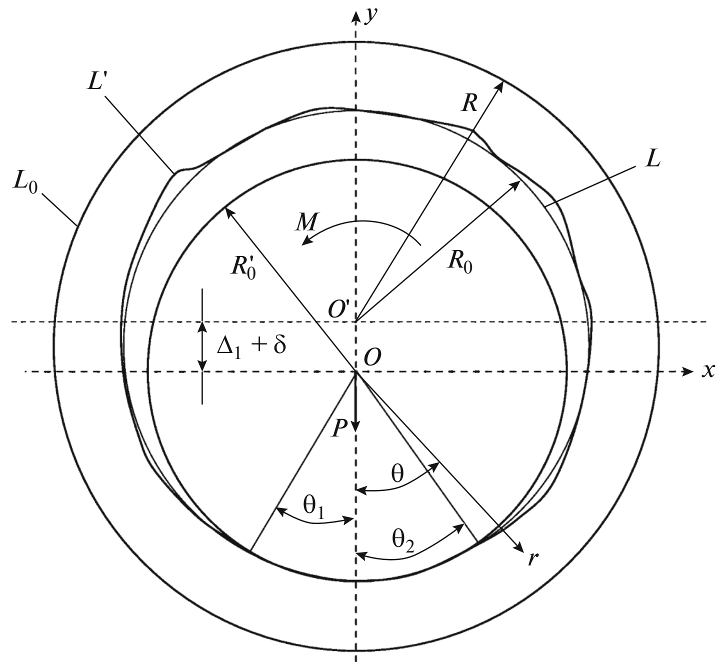

Fig. 1. Design diagram of contact problem for hub–plunger frictional pair.

isfy, as well as the kinetic equation of the abrasive wear of the material of the frictional pair, as the mathematical model.

To find the contact pressure, distribution of the temperature, and stresses in the elements of the friction unit, we need to consider the wear-and-contact problem on the indentation of the plunger in the internal hub surface [7].

Let the plunger be pressed to the internal hub surface at some part of its section. We assume that there are some displacements in the points of the internal hub surface. We believe that the conditions for the deformation of a plane are fulfilled and the operating modes of the frictional pair when the residual strains can occur are inadmissible. It is considered that the loading conditions are quasi static.

We will assume that the hub is in the polar coordinate system $r\theta$ and choose the coordinate origin at the center of the concentric circles L_0 (with radius R_0) and L (with radius R). It is believed that the internal contour L'_0 of the hub is close to circular. Let its unknown boundary have the form

$$r = \rho(\theta), \quad \rho = R_0 + \varepsilon H(\theta),$$

$$H(\theta) = \sum_{k=0}^{\infty} (a_k^0 \cos k\theta + b_k^0 \sin k\theta),$$

where $\varepsilon = R_{\max}^0 / R_0$ is the small parameter and $R_{\max}^0$ is the maximum height of the lug (hollow) of the irregularity of the circle profile $r = R_0$.

In addition, the unknown in advance external contour of the plunger is close to circular and can be represented as

$$\rho_1(\theta) = R'_0 + \varepsilon H_1(\theta),$$

$$H_1(\theta) = \sum_{k=0}^{\infty} (a_k^1 \cos k\theta + b_k^1 \sin k\theta),$$

where $H_1(\theta)$ needs to be determined in the course of solving the optimization problem.

Without losing generality of the optimization problem under consideration, we assume that function $H(\theta)$ and $H_1(\theta)$ can be expressed as a Fourier series.

The main contact condition connecting the hub and plunger displacements has the form [7, 8]

$$\upsilon_1 + \upsilon_2 = \delta(\theta), \quad \theta_1 \le \theta \le \theta_2. \tag{1}$$

Here $\delta(\theta)$ is the immersion of the points of the hub and plunger surface, which is determined by the form of the internal surface of the hub and plunger and the magnitude of the pressing force P and (θ_2–θ_1) is the angle (pad) of the contact of the hub and plunger.

In the contact zone, in addition to the normal pressure $p(\theta,t)$, the shearing stress related to the contact pressure according to the Coulomb–Amonton law is also present. The contact pressure $p(\theta)$ needs to be determined in the course of solving the optimization problem.

The shearing forces (friction forces) cause the heat release in the contact zone; moreover, the total amount of heat in unit time is proportional to the power of the frictional forces, and the heat quantity which is released at point θ of the contact zone is

$$Q(\theta,t) = Vfp(\theta,t),$$

where V is the period average speed of the plunger's movement relative to the hub and f is the coefficient of friction of the frictional pair.

The total heat quantity $Q(\theta,t)$ will be spent on the heat flux into the hub $Q_b(\theta,t)$ and the heat flux $Q_1(\theta,t)$, raising the temperature of the plunger. Since the frequency of the movement of the plunger is sufficiently large, we will consider the problem of determining the temperature as a stationary problem.

For displacements of the points of the friction surface of the hub and plunger, we have

$$\upsilon_1 = \upsilon_{1e} + \upsilon_{1r} + \upsilon_{1w}, \quad \upsilon_2 = \upsilon_{2e} + \upsilon_{2r} + \upsilon_{2w},$$

where υ_{1e}, υ_{2e} are the thermoelastic displacements of the points of the contact surface of the hub and plunger, respectively; υ_{1r} and υ_{2r} are the displacements caused by the crumpling of the microirregularities of the hub and plunger, respectively; and υ_{1w} and υ_{2w} are the displacements caused by the wear of the hub and plunger surfaces, respectively.

The rate of change of the displacements of the surface during the abrasive wear of the frictional pair is [7, 8]

$$\frac{d\upsilon_{kw}}{dt} = K^{(k)}p(\theta,t), \tag{2}$$

where $K^{(k)}$ is the coefficient of the wear of the hub and plunger material ($k = 1, 2$), respectively.

In order to determine the displacements υ_{1e}, υ_{1r} and υ_{2e}, υ_{2r}, we need to solve the problems of thermoelasticity for the hub and plunger. For the hub we have:

$$\Delta T = 0, \quad \text{at} \quad r = \rho(\theta),$$

$$A_{T_1}\lambda\frac{\partial T}{\partial n} - A_{T_2}\alpha_1 T = -Q_*,$$

$$Q_*(\theta) = \begin{cases} Q_b(\theta) & \text{on the contact pad,} \\ 0 & \text{outside the contact pad,} \end{cases}$$

$$\text{at} \quad r = R, \quad \lambda\frac{\partial T}{\partial r} + \alpha_2 T = 0;$$

$$\text{at} \quad r = \rho(\theta), \quad \sigma_n = -p(\theta),$$

$$\tau_{nt} = -fp(\theta) \quad \text{on the contact pad;}$$

$$\sigma_n = 0, \quad \tau_{nt} = 0 \quad \text{outside the contact pad;}$$

$$\text{and} \quad \text{at} \quad r = R, \quad \upsilon_r - i\upsilon_\theta = g(\theta).$$

Here Δ is the Laplace operator; T is the excess temperature of the hub; α_1 and α_2 are the heat-transfer coefficients from the internal and external hub surfaces, respectively; n and t are the normal and tangent to the internal contour of the hub; A_{T_1} is the heat-absorbing surface; A_{T_2} is the cooling surface; λ is the coefficient of thermal conductivity of the hub material; Q_* is part of the heat which is released during friction and is due to the heating of the hub; υ_r and υ_θ are the radial and tangential components of the displacement vector of the points of the hub contour L, respectively; $g(\theta)$ is the desired function of the displacements of the point of the external contour of the hub; σ_n, σ_t, and τ_{nt} are components of the stress tensor; and $i^2 = -1$.

Similarly, the thermoelasticity problem is posed to determine the displacements υ_{2e} and υ_{2r} of the contact surface of the plunger:

on the contact surface,

$$\Delta T_1 = 0 \quad \text{at} \quad r = \rho_1(\theta), \quad \lambda_1\frac{\partial T_1}{\partial n} = -Q_1(\theta);$$

outside the contact pad,

$$\lambda_1\frac{\partial T_1}{\partial n} + \alpha_1 T_1 = 0;$$

on the contact pad,

$$\sigma_n = -p(\theta), \quad \tau_{nt} = -fp(\theta);$$

$$\sigma_n = 0, \quad \tau_{nt} = 0,$$

outside the contact pad.

To determine υ_{1w} and υ_{2w} kinetic equation (2) of the wear of the hub and plunger is used. For the solution of the posed problem, it is necessary to jointly solve the wear-and-contact problem on the indentation of the plunger into the internal hub surface and the problem of minimizing the thermal state of the hub.

Values θ_1 and θ_2 (the ends of the contact section of the plunger with the hub) are unknown. To define them we use the condition [9] that pressure $p(\theta)$ continuously goes to zero when point θ goes beyond the contact section

$$p(\theta_1) = 0, \quad p(\theta_2) = 0. \tag{3}$$

To define the displacement function $g(\theta)$ of the points of the external hub contour and microgeometry of the friction surface, the problem statement needs to be supplemented with the condition (criterion) which makes it possible to define the desired function $g(\theta)$, $H(\theta)$, and $H_1(\theta)$. We take the temperature distribution at the contact surface that is close to uniform as such a criterion.

Without losing the generality of the stated problem, we assume that the desired function $g(\theta)$ of the displacements of the points of the external contour is expanded in a Fourier series. Consequently, the Fourier coefficients of functions $g(\theta)$, $H(\theta)$, and $H_1(\theta)$ should ensure the uniform temperature distribution on the contact surface. This additional condition makes it possible to define the desired function $g(\theta)$, $H(\theta)$, and $H_1(\theta)$.

SOLUTION METHOD

The temperature functions, stresses, and displacements in the hub and plunger, as well as the contact pressure we find in the form of small parameter expansions, in which for simplicity we neglect the terms that contain ε, with degrees higher than the first degree. Each of the approximations satisfies the set of differential equations of plain thermoelasticity. We will obtain the values of the temperature, components of the stress tensor, and displacement vector at $r = \rho(\theta)$ (similarly to $r = \rho_1(\theta)$) when expanding the expressions for temperature, stresses, and displacements in the neighborhood of $r = R_0$ into series. Using the perturbation method and boundary conditions of the problem, we obtain the sequence of boundary problems of the thermoelasticity theory for the hub and plunger.

The solution of the boundary-value problem of the theory of thermal conductivity is sought by the method of the separation of variables.

To solve the thermoelasticity problem in each approximation, we use the themoelastic potential of displacements $F^{(0)}(r,\theta)$. After defining this potential for the hub in a zero-order approximation, we calculate stresses $\bar{\sigma}_r^{(0)}$, $\bar{\sigma}_\theta^{(0)}$, and $\bar{\tau}_{r\theta}^{(0)}$ and displacements $\bar{\upsilon}_r^{(0)}$ and $\bar{\upsilon}_\theta^{(0)}$ in the hub by using the well-known formulas [10]. The found stresses and displacements will not satisfy the boundary conditions of a zero-order approximation. Thus, for the hub, we need to find such a second stress-strain state $\bar{\bar{\sigma}}_r^{(0)}$, $\bar{\bar{\sigma}}_\theta^{(0)}$, $\bar{\bar{\tau}}_{r\theta}^{(0)}$, $\bar{\bar{\upsilon}}_r^{(0)}$, $\bar{\bar{\upsilon}}_\theta^{(0)}$ so that the boundary conditions of the zero-order approximation is fulfilled.

The boundary conditions of the problem using the Kolosov–Muskhelishvili formulas [9] can be written as a boundary problem to find two complex potentials $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$ for the hub. The complex potentials $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$ are sought in the form

$$\Phi^{(0)}(z) = \sum_{k=-\infty}^{\infty} a_k z^k, \quad \Psi^{(0)}(z) = \sum_{k=-\infty}^{\infty} b_k z^k.$$

Using the power series method, we find for coefficients a_k and b_k ($k = 0, \pm1, \pm2, \ldots$) an infinite linear algebraic set of equations, the solution of which does not present special difficulties (see [9], §59). The right side of these formulas contain coefficients α_k and β_k of the expansions of the contact pressure $p^{(0)}(\theta)$ and the desired function

$$g^{(0)\prime}(\theta) = \sum_{k=-\infty}^{\infty} (a_k^{*0} \cos k\theta + b_k^{*0} \sin k\theta).$$

Using the complex potentials $\Phi^{(0)}(z)$ and $\Psi^{(0)}(z)$, the Kolosov-Muskhelishvili formulas and integration of the kinetic equation (2) of the wear of the hub material, we define the displacement $\upsilon_1^{(0)}$ of the contact surface of the hub in the zero-order approximation. Similarly, we find the solution of the thermoelasticity problem in the zero-order approximation for the plunger. We substitute the found amounts $\upsilon_1^{(0)}$ and $\upsilon_2^{(0)}$ in the basic contact equation (1) in the zero-order approximation. For the algebraization of the basic contact equation (1), we try to find the unknown functions of the contact pressure in the zero-order approximation in the form of expansions. Substituting these expansions in Eq. (1) in the zero-order approximation, we obtain the functional equations for the sequential definition of $p_0^0(\theta), p_1^0(\theta)$, etc. In order to construct the algebraic set of equations and find α_k and β_k, we equate the coefficients in the same trigonometric functions in the left and right sides of the functional equation of the contact problem and then obtain the infinite algebraic set of equations relative to α_k^0 ($k = 0, 1, 2,\ldots$), β_k^0 ($k = 1, 2,\ldots$) and α_k^1, β_k^1, etc., which due to the unknown values θ_1 and θ_2 ($\theta_1 = \theta_1^0 + \varepsilon\theta_1^1 + \ldots$; $\theta_2 = \theta_2^0 + \varepsilon\theta_2^1 + \ldots$) turns out to be nonlinear. To determine values θ_1 and θ_2 we have condition (3), which can be represented as follows: for a zero-order approximation, $p^{(0)}(\theta_1^0) = 0$ and $p^{(0)}(\theta_2^0) = 0$; for a first-order approximation, $p^{(1)}(\theta_1^1) = 0$ and $p^{(1)}(\theta_2^1) = 0$.

The right sides of the obtained algebraic sets of equations in a zero-order approximation contain unknown coefficients a_k^{*0} and b_k^{*0} of the expansion of function $g^{(0)}(\theta)$. To construct the missing equations of the set of equations, we use the criterion (principle) of the uniform temperature distribution on the contact surface. Let $\bar{T} = \bar{T}_0 + \varepsilon\bar{T}_1 + \ldots$ be the optimal value of the temperature on the friction surfaces. The value of $\bar{T}$ is unknown in advance and subject to determination in the course of solving the optimization problem.

The distribution of the excess temperature on the internal hub surface in the zero-order approximation is of the form

$$T_{\text{surf}}^{(0)}(\theta) = T_{|r=R}^{(0)}. \tag{4}$$

The obtained formula (4) shows that the temperature linearly depends on coefficients a_k^{*0} and b_k^{*0} of the Fourier series of the function of the displacement $g^{(0)}(\theta)$.

For the temperature of the hub surface, we can write

$$T^{(0)}(\theta,\tau) = F_0(\theta,\tau,a_0^{*0},a_k^{*0},b_k^{*0}), \tag{5}$$

where $k = 1, 2, ..., m$; $T^{(0)}(\theta, \tau)$ is the function of the independent variable θ and $2m + 1$ parameters a_0^{*0}, a_1^{*0}, ..., a_m^{*0} and b_1^{*0}, ..., b_m^{*0}; τ is time; we consider it a free parameter; and a_0^{*0}, a_k^{*0}, and b_m^{*0} are constant parameters (time-dependent); however, they are not known in advance and subject to determination.

The problem which is stated here is to find such values of the parameters that ensure function (5) of the temperature of friction surface is a constant value in the best way.

According to the least squares principle, the most probable values of the parameters in a zero-order approximation will be such values at which the sum of the squares of the deviations will be the minimal values, i.e.,

$$U_0 = \sum_{i=1}^{M} [F_0(\theta_i, t, a_0^{*0}, a_k^{*0}, b_k^{*0}) - \bar{T}_0]^2 \to \min. \quad (6)$$

For any point in time, values a_0^{*0}, a_k^{*0}, b_m^{*0} $(k = 1, 2, ..., m)$, and $\bar{T}_0$ are considered as independent variables. Equating to zero the partial derivatives of the left side of expression (6) over variables a_0^{*0}, a_k^{*0}, b_m^{*0}, and $\bar{T}_0$, we obtain $2m + 2$ equations with $2m + 2$ unknown quantities

$$\frac{\partial U}{\partial a_0^{*0}} = 0, \quad \frac{\partial U}{\partial a_k^{*0}} = 0, \quad (k = 1, 2, \ldots, m) \quad (7)$$

$$\frac{\partial U}{\partial b_k^{*0}} = 0, \quad \frac{\partial U}{\partial \bar{T}_0} = 0.$$

Function $F_0(\theta, \tau, a_0^{*0}, a_k^{*0}, b_k^{*0})$ is linear relative to the unknown parameters; thus, generation of the set of equations (7) is greatly simplified and it can be written as a normal set of equations. Adding the linear set of equations (7) to the infinite set of equations of the wear-and-contact problem in a zero-order approximation, we obtain the closed set of equations for the desired coefficients of the optimization problem. To solve it in a zero-order approximation, we used the methods of reduction and successive approximations [11].

After defining the desired quantities of the zero-order approximation, we proceed to construct the solution of the problem in a first-order approximation. Based on the obtained solution at $r = R_0$, we define functions N and T_t [4]. The further course of the solution is similar to that of a zero-order approximation. The right side of the set of equations for the determination of quantities $a_k^{(1)}$ and $b_k^{(1)}$ contains the desired coefficients of the expansion of the contact pressure $p^{(1)}(\theta)$ and the displacement function

$$g^{(1)\prime}(\theta) = \sum_{k=-\infty}^{\infty} (a_k^{*1} \cos k\theta + b_k^{*1} \sin k\theta).$$

The thermoelasticity problem for a plunger is solved in the first-order approximation similarly.

We algebraize the resolving equation of the contact problem (1) in the first-order approximation as in the zero-order approximation. Because of the unknown quantities θ_1^1 and θ_2^1 the set of equations turns out to be nonlinear. The constructed set of equations in each approximation is not closed because of the unknown function $g^{(1)\prime}(\theta)$, $H(\theta)$, and $H_1(\theta)$.

Given functions $g^{(0)\prime}(\theta)$, $g^{(1)\prime}(\theta)$, $H(\theta)$, and $H_1(\theta)$ and using numerical calculations, the obtained set of equations in the zero-order and in the first-order approximations makes it possible to define the contact pressure, the temperature distribution, the stress-strain state, and the wear of the hub and plunger of the frictional pair.

The right sides of the obtained algebraic sets of equations of the wear-and-contact problem in the first-order approximation contain the unknown coefficients a_k^{*1} and b_k^{*1} of the expansion of function $g^{(1)}(\theta)$ and a_k^0, b_k^0, a_k^1, and b_k^1 of functions $H(\theta)$ and $H_1(\theta)$.

In order to construct the missing equations, we need the temperature distribution on the contact surface to be uniform in the first-order approximation. The distribution of the excess temperature on the internal hub surface $T_{\text{surf}}^{(1)}(\theta)$ is of the form

$$T_{\text{surf}}^{(1)}(\theta) = \left[\frac{\partial T^{(0)}}{\partial r} H(\theta) + T^{(1)}(r, \theta)\right]_{r=R}. \quad (8)$$

The obtained formula for the temperature of friction surface of the hub in the first-order approximation (8) shows that the temperature linearly depends on coefficients a_k^*, b_k^*, a_k^0, b_k^0, a_k^1, and b_k^1. For the temperature of the friction hub surface in the first-order approximation, we can write

$$T^{(1)}(\theta, \tau) = F_1(\theta, \tau, a_0^{*1}, a_k^{*1}, b_k^{*1}, a_0^0, a_k^0, b_k^0, a_0^1, a_k^1, b_k^1),$$

where $k = 1, 2, ..., m$; $T^{(1)}(\theta, \tau)$ is a function of the independent variable θ and $6m + 3$ parameters a_0^{*1}, a_1^{*1}, ..., a_m^{*1}, b_1^{*1}, ..., b_m^{*1}, a_0^{*0}, a_0^0, a_1^0, ..., a_m^0, b_1^0, ..., b_m^0, a_0^1, a_1^1, ..., a_m^1, b_1^1, ..., b_m^1.

Using the least squares principle, we have

$$U_1 = \sum_{i=1}^{M} [F_1(\theta, \tau, a_0^{*1}, a_k^{*1}, b_k^{*1}, a_0^0, a_k^0, \\ b_k^0, a_0^1, a_k^1, b_k^1) - \bar{T}_1]^2 \to \min. \quad (9)$$

Equating to zero the partial derivatives of the left side for these variables for any point in time, we obtain exactly $6m + 4$ equations with $6m + 4$ unknown quantities. Adding this linear set of equations to the infinite set of equations of the wear-and-contact problem in

Table 1. Values of Fourier coefficients of functions $g(\theta)$ ad microgeometry of friction surface $H(\theta)$, $H_1(\theta)$

a_0^*	a_1^*	a_2^*	a_3^*	a_4^*	a_5^*	b_1^*	b_2^*	b_3^*	b_4^*	b_5^*
0.124	0.071	0.048	0.032	0.016	0.012	0.0121	0.050	0.025	0.013	0.009
a_2^*	a_1^0	a_2^0	a_3^0	a_4^0	a_5^0	b_1^0	b_2^0	b_3^0	b_4^0	b_5^0
0.722	0.631	0.579	0.345	0.206	0.161	0.708	0.506	0.368	0.257	–0.164
a_0^1	a_1^1	a_2^1	a_3^1	a_4^1	a_5^1	b_1^1	b_2^1	b_3^1	b_4^1	b_5^1
0.354	0.229	0.193	0.0175	0.097	0.072	0.343	0.218	0.195	0.112	–0.067

the first-order approximation, similarly to the zero-order approximation, we obtain the closed algebraic set of equations to determine the coefficients $\alpha_{0,0}^1$, $\alpha_{k,0}^1, \beta_{k,0}^1, \alpha_{0,1}^1, \alpha_{k,1}^1, \beta_{k,1}^1, \theta_1^1, \theta_2^1, a_0^{*1}, a_k^{*1}, b_k^{*1}, a_0^0, a_k^0, b_k^0$, a_0^1, a_k^1, and b_k^1.

ANALYSIS OF SIMULATION RESULTS

To solve the nonlinear algebraic set of equations, we used the methods of reduction and successive approximations. In each approximation, the unified set of equations was solved using the Gauss method with the selection of the main element for the example of the hub-plunger pair of the slush pump of the U8-6MA2 type. The calculation results for the determination of the function of displacements of the points of the external hub contour (the values of coefficients a_k^* and b_k^* are in millimeters) and of the microgeometry of the friction surface (the values of coefficients a_k^0, b_k^0, a_k^1, and b_k^1 are in micrometers) with the plunger's motion speed $V = 0.2$ m/s are presented in Table 1.

The calculations showed that the value of temperature $\bar{T}$ at the optimal function of displacements of the points of the external hub contour and microgeometry of the friction surface is reduced by factors of 1.12 to 1.20 compared with the friction units which are commonly used for the slush pumps. The proposed method for the optimization of the thermal state in the hub-plunger frictional pair can be extended to the construction of other friction units.

It should be noted that we can choose in advance the value of $\bar{T}$ from the condition of ensuring the heat resistance of the frictional pair. However, as the calculations show, in this case, the sums of the squares of the deviations turn out to be more significant. With the help of finding the unknown optimal value of the temperature $\bar{T}$, the sums of the squares of the deviations decrease; i.e., the search results turn out to be more accurate.

CONCLUSIONS

An effective method to reduce the temperature of the hub of a frictional pair by the criterion of a uniform temperature distribution on the friction surface at the design stage is proposed. The calculations show that the value of the temperature at the optimal function of the displacement of the points of the external hub contour and microgeometry of the friction surface decreases by factors of 1.12 to 1.20 compared with the friction units which are commonly used for slush pumps. Thus, by choosing the functions of the displacements of the points of the external hub contour and microgeometry of the friction surface of the frictional pair, it is possible to control (minimize) the reduction of the thermal state of the materials in the friction unit. Knowledge of the optimal coefficients a_k^*, b_k^*, a_k^0, b_k^0, a_k^1, and b_k^1 of functions $g(\theta)$, $H(\theta)$, and $H_1(\theta)$ makes it possible to ensure the improved performance of the frictional pair at the recursive-forward movement of the plunger at the stage of design and manufacture.

NOTATION

$p(\theta, t)$	the contact pressure
ε	the small parameter
$H(\theta)$	the desired function describing the microgeometry of the internal hub surface
$H_1(\theta)$	the desired function describing the microgeometry of the plunger surface
R and R_0	the radii of the circles of contours L and L_0, respectively
$g(\theta)$	the desired function of displacements of the points of the external hub contour
a_k^0 and b_k^0	the coefficients of the Fourier series of function $H(\theta)$
t	the time
a_k^1 and b_k^1	the coefficients of the Fourier series of function $H_1(\theta)$

$\rho(\theta)$	the function describing the internal hub surface
f	the friction coefficient of the contact pair
α_k^0, β_k^0, $\alpha_{k,0}^0$, $\beta_{k,0}^0$, α_k^1, β_k^1, $\alpha_{k,1}^0$, and $\beta_{k,1}^0$	the coefficients of the Fourier series for the functions defining the contact pressure
V	the period average speed of displacement of the shaft relative to the hub
λ	the coefficient of heat conductivity of the hub material
$T(r,\theta)$	the temperature function
a_k^{*0}, b_k^{*0}, a_k^{*1}, and b_k^{*1}	the coefficients of the Fourier series of the desired function of displacements of the points of the external hub contour

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