Metadata of the chapter that will be visualized in SpringerLink

Book Title	13th International Conference on Theory and Application of Fuzzy Systems and Soft Computing — ICAFS 2018
Series Title	
Chapter Title	Decision Making in Investment Problem by Using Self-confidence Based Preference Relation
Copyright Year	2019
Copyright HolderName	Springer Nature Switzerland AG

Corresponding Author	Family Name	**Jabbarova**
	Particle	
	Given Name	**Aynur I.**
	Prefix	
	Suffix	
	Role	
	Division	
	Organization	Azerbaijan State Economic University
	Address	Baku, Azerbaijan
	Email	stat_aynur@mail.ru

Abstract	In this paper we use interval-valued preference relations with self-confidence for investment problem. For calculating priority vectors of this preference relations linear programming are used. We use TOPSIS method the same problem for check results first method.

Keywords (separated by '-')	Linear programming - Self-confidence levels - Priority vector - TOPSIS method

Decision Making in Investment Problem by Using Self-confidence Based Preference Relation

Aynur I. Jabbarova(✉)

Azerbaijan State Economic University, Baku, Azerbaijan
stat_aynur@mail.ru

Abstract. In this paper we use interval-valued preference relations with self-confidence for investment problem. For calculating priority vectors of this preference relations linear programming are used. We use TOPSIS method the same problem for check results first method.

Keywords: Linear programming · Self-confidence levels · Priority vector TOPSIS method

1 Introduction

The priorities of real issues are indefinite. Different types of investigations have been conducted to determine this diversity in decision models. In general, these studies can be classified in the following areas. (1) incomplete priorities; (2) models of confidence; (3) fuzzy priority relations with partial trust.

Incomplete priority models can make obstacles to finding the right choices because of the lack of information and the complexity of alternatives. On the other hand, this indecision leads to failure of the group of decision-makers does not come to a conclusion. Formally, the alternatives $\mathcal{A}$, there exists $f, g \in \mathcal{A}$ such that neither $f \succeq g$ nor $g \succeq f$ is presumed [1]. For modelling of incomplete advantages, the vector advantages [1], indefinite probabilities [2–5], indefinite advantage and other approaches are offered [6, 7].

Fuzzy priority relations are used in cases where the decision-maker cannot prefer one of the alternatives due to complexity of alternatives, lack of knowledge and information and other factors. The advantages are of "distributed" nature to reflect that an alternative is better than the other one. Unlike the classical advantage relations, fuzzy priority relations (FPR) reflects that $\tilde{f}$ alternative is more advantageous than $\tilde{g}$ alternative in comparison of $\tilde{g}'$ alternative with $\tilde{f}'$ alternative.

In [8], new priority model is oferred. This model enables to define the priority degree given with the self-confidence level. The self-confidence level describes the confidence of the decision-maker in fuzzy priority. This approach is the best mean when it has the fuzzy priorities in [9–12] and indefinite priorities in [13–16].

R. A. Aliev et al. (Eds.): ICAFS 2018, AISC 896, pp. 1–7, 2019.
https://doi.org/10.1007/978-3-030-04164-9_108

2 Preliminaries

Fuzzy Preferences with Self-confidence Level. In real-world problems, a DM may not be completely sure in his preferences. In such cases, FPR is assigned by a self-confidence level described by a linguistic term form a predefined codebook. An FPR with self-confidence level proposed in [8] is described as follows.

Definition 1 [8]. FPR with Self-confidence Level. Let $R : \mathcal{A} \times \mathcal{A} \to T$ be a fuzzy preference relation with self-confidence based on a finite set of alternatives $\mathcal{A}$ shown as follows,

$$R = ((r_{ij}, s_{ij})) \tag{1}$$

where r_{ij} denotes the degree or intensity of preference of alternative $\tilde{f}_i$ over alternative $\tilde{f}_j$, and s_{ij} represents the self-confidence level on the preference value r_{ij}. It is assumed that $r_{ij} + r_{ji} = 1, s_{ij} = s_{ji}$ [8].

Consistency of an FPR with the self-confidence level is considered in terms of transitivity properties [8]. They consider weak stochastic transitivity, strong stochastic transitivity and additive transitivity at a confidence level s. These properties are considered as those of common FPR, but satisfied at some lowest possible self-confidence level.

The FPR with the self-confidence level [8] is a new step in development of a decision theory. It encompasses both a degree of preference and the related belief level. However, this approach is of two main shortcomings: the degree of preference is crisp and, what is more important, an essence of self-confidence level is not considered. However, a self-confidence level is naturally of a probabilistic character and may be considered as a fuzzy value of a probability measure of a fuzzy degree of preference. In this report, we propose a Z-valued preference relation as a more general preference model.

Definition 2 [17]. Comparison of intervals. The degree to which $[\underline{I}, \overline{I}]$ is higher than $[\underline{J}, \overline{J}]$ is defined as follows.

$$d(I,I) = \begin{cases} \dfrac{\overline{I} - \overline{J}}{(\overline{I} - \overline{J}) + (\underline{J} - \underline{I})}, & \overline{I} > \overline{J}, \quad \underline{J} \geq \underline{I} \\ 1, & \overline{I} = \overline{J}, \quad \underline{J} > \underline{I} \\ & or \quad \overline{I} > \overline{J}, \quad \underline{I} \geq \overline{J} \\ & or \quad \overline{I} = \overline{J}, \quad \underline{I} = \overline{J} \\ 1 - d(I,J), & otherwise \end{cases}$$

3 Statement of the Problem and a Solution Method

At first we applied self-confidence based preference relation method to our investment problem. A company is planning to make an investment in three sphere; A1-agriculture, A2-processing industry, A3-tourism sector/Each alternative is characterized by 3 criteria; C1-volume of income, C2-degree of risk, C3-enviromental impact. The codebook for interval-valued level is given in Table 1. AQ1

Table 1. The codebook for interval-valued confidence level

	Interval value
Medium	[0.4 0.6]
Medium high	[0.6 0.8]
High	[0.7 0.9]
Very high	[0.9 1]

For calculating we comprised of intervals by using Definition.

$$d(VH, MH) = 1$$

$$d(VH, H) = 1$$

$$d(H, MH) = 1$$

Next, we offer 3 × 3 fuzzy preference relation with interval-valued self-confidence:

$$P = \begin{pmatrix} (0.5, VH) & (0.7, MH) & (0.9, H) \\ (0.3, MH) & (0.5, VH) & (0.7, H) \\ (0.1, H) & (0.3, H) & (0.5, VH) \end{pmatrix}.$$

We use the linear programming model for determine priority vector of P:
Objective function

$$\min z = z_{12} + z_{13} + z_{23}$$

subject to

$$\begin{cases} 0.5w_1 - 0.5w_2 - y_{12} = 0.2, \\ 0.5w_1 - 0.5w_3 - y_{13} = 0.4, \\ 0.5w_2 - 0.5w_3 - y_{23} = 0.2, \\ z_{12} - 2y_{12} \geq 0, \\ z_{12} + 2y_{12} \geq 0, \\ z_{13} - 3y_{13} \geq 0, \\ z_{13} + 3y_{13} \geq 0, \\ z_{23} - 3y_{23} \geq 0, \\ z_{23} + 3y_{23} \geq 0, \\ w_1 + w_2 + w_3 = 1, \\ w_i \geq 0, \quad i = 1, 2, 3 \\ z_{ij} \geq 0, \quad i, j = 1, 2, 3 \end{cases}$$

We solve this problem and find $z = 0.3$ and priority vector $w = (0.7, 0.3, 0)$. This results show that 1st alternative is best alternative. Then we use TOPSIS method [17] for solving this problem and compare with below method.

Importance weights of criteria: $w_1 = [0.4 - 0.5]$, $w_2 = [0.3 - 0.35]$, $w_3 = [0.15 - 0.3]$.

Decision matrix for investment problem is given in Table 2. AQ2

Table 2. Decision matrix

	C_1	C_2	C_3
A_1	8	2	3
A_2	6	5	4
A_3	3	7	7

1. Calculate the normalized decision matrix by using following formula (Table 3):

Table 3. Normalized decision matrix

	C_1	C_2	C_3
A_1	0.77	0.23	0.35
A_2	0.57	0.57	0.47
A_3	0.29	0.79	0.81

$$n_{ij} = \frac{c_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}}, \; j = 1, \ldots, m, \; i = 1, \ldots, n.$$

Table 4. Weighted normalized decision matrix]

	C_1	C_2	C_3
A_1	[0.308 0.385]	[0.069 0.0805]	[0.0525 0.105]
A_2	[0.228 0.285]	[0.171 0.1995]	[0.0705 0.141]
A_3	[0.116 0.145]	[0.237 0.2765]	[0.1215 0.243]

Table 5. Positive and negative ideal solutions

	C_1	C_2	C_3
Positive ideal	[0.385]	[0.2765]	[0.243]
Negative ideal	[0.116]	[0.069]	[0.0525]

2. Calculate the weighted normalized decision matrix $r_{ij} = n_{ij} \cdot w_i.$, where $j = 1, \ldots, m$, $i = 1, \ldots, n$ and $\sum_{i=i}^{n} w_i = 1$ (Table 4).
3. Calculate the positive and the negative ideal solution by using following formula (Table 5):

$$A^+ = \{r_1^+, \ldots, r_n^+\} = \left\{ \left(\max_j r_{ij} \middle| i \in I \right) \right\},$$
$$A^- = \{r_1^-, \ldots, r_n^-\} = \left\{ \left(\min_j r_{ij} \middle| i \in I \right) \right\}.$$

4. Determine the separation measures, using n-dimensional Euclidean distance. The calculated separation of each alternative from the positive ideal solution by using following formula

$$d_j^+ = \left\{ \sum_{i=1}^{n} \vec{r}_{ij}^L - \bar{r}_i^- \right\}^{\frac{1}{2}}, \tag{3}$$

A_1 $(0.308 - 0.385)^2 + (0.069 - 0.2765)^2 + (0.0525 - 0.243)^2 = 0.077^2 + 0.2075^2 + 0.1905^2 = 0.005929 + 0.043056 + 0.03629 = 0.085275;$

A_2 $(0.228 - 0.385)^2 + (0.171 - 0.2765)^2 + (0.0705 - 0.243)^2 = 0.157^2 + 0.1055^2 + 0.1725^2 = 0.024649 + 0.01113 + 0.029756 = 0.065535;$

A_3 $(0.116 - 0.385)^2(0.237 - 0.2765)^2 + (0.1215 - 0.243)^2 = 0.269^2 + 0.0395^2 + 0.1215^2 = 0.072361 + 0.00156 + 0.014762 = 0.088683;$

The calculated separation of each alternative from the negative ideal solution by following formula

$$d_j^- = \left\{ \sum_{i=1}^{n} \vec{r}_{ij}^U - \bar{r}_i^+ \right\}^{\frac{1}{2}}. \quad (4)$$

A_1 $(0.385 - 0.116)^2 + (0.0805 - 0.069)^2 + (0.105 - 0.0525)^2 =$
$0.269^2 + 0.0115^2 + 0.0525^2 = 0.072361 + 0.000132 + 0.002756 = 0.075249;$
A_2 $(0.285 - 0.116)^2 + (0.1995 - 0.069)^2 + (0.141 - 0.0525)^2 = 0.169^2$
$+ 0.1305^2 + 0.0885^2 = 0.028561 + 0.01703 + 0.007832 = 0.053423;$
A_3 $(0.145 - 0.116)^2 +(0.2765 - 0.069)^2 + (0.243 - 0.0525)^2 = 0.029^2$
$+ 0.2075^2 + + 0.1905^2 = 0.000841 + 0.043056 + 0.03629 = 0.080187$

5. The calculated the relative measures by using (5)

$$R_j = \frac{d_j^-}{(d_j^+ - d_j^-)}, j = 1, \ldots, m, \quad (5)$$

$$R_1 = \frac{0.075249}{0.075249 + 0.085275} = \frac{0.075249}{0.160524} \approx 0.469,$$
$$R_2 = \frac{0.053423}{0.065535 + 0.053423} = \frac{0.053423}{0.118958} \approx 0.45,$$
$$R_3 = \frac{0.080187}{0.088683 + 0.080187} = \frac{0.080187}{0.16887} \approx 0.475.$$

The ranking of relative measures the preference order are given in Table 6. AQ3

Table 6. The ranking of relative measure

Alternatives	R_j	Rank
A_1	0.469	3
A_2	0.45	2
A_3	0.475	1

The results represent that alternative A_3 is the best alternative. This result significantly differ from the result obtained by the self-confidence based preference relations. The reason is that information on DM's confidence level on assigned preference is disregarded. As one can see, this may lead to choice of a non-optimal alternative.

4 Conclusion

In this article, the issue of capital investment has been solved through a method based on interval-value fuzzy priority. This method is characterized by the self-confidence level that the decision maker has given to alternatives in advance. The issue has been solved through linear programming and has been assigned a priority vector. The issue has been solved through linear programming and has been assigned a priority vector.

Then this issue was solved by the TOPSIS algorithm and the best alternative was set. The results obtained through both methods have been analyzed and the results obtained by the first method have been shown to be more adequate.

References

1. Ok, E.A.: Utility representation of an incomplete preference relation. J. Econ. Theory **104**, 429–449 (2002)
2. Nau, R.: The shape of incomplete preferences. Ann. Stat. **34**, 2430–2448 (2006)
3. Insua, D.R.: On the foundations of decision making under partial information. Theory Decis. **33**, 83–100 (1992)
4. Rigotti, R., Shannon, C.: Uncertainty and risk in financial markets. Econometrica **73**, 203–243 (2005)
5. Walley, P.: Statistical inferences based on a second-order possibility distribution. Int. J. Gen Syst **9**, 337–383 (1997)
6. Aliev, R.A.: Fuzzy Knowledge based Intelligent Robots. (Radio i Svyaz, Moscow) (in Russian) (1995)
7. Aliev, R.A., Huseynov, O.H.: A new approach to behavioral decision making with imperfect information. In: Proceedings of 6th International Conference on Soft Computing and, Computing with Words in System Analysis, Decision and Control, ICSCCW, pp. 227–237 (2011)
8. Liu, W., Dong, Y., Chiclana, F., Herrera-Viedma, E., Cabrerizo, F.J.: A new type of preference relations: fuzzy preference relations with self-confidence. In: IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), pp. 1677–1684 (2016)
9. Blin, J.M., Whinston, A.B.: Fuzzy sets and social choice. J. Cybern. **3**(4), 17–22 (1973)
10. Dubois, D., Prade, H.: A review of fuzzy sets aggregation connectives. Inf. Sci. **36**, 85–121 (1985)
11. Herrera-Viedma, E., Herrera, F., Chiclana, F.: Multiperson decision-making based on multiplicative preference relations. Eur. J. Oper. Res. **129**(2), 372–385 (2001)
12. Herrera, F., Herrera-Viedma, E.: Aggregation operators for linguistic weighted information. IEEE Trans. Syst. Man Cybern. A **27**, 646–656 (1997)
13. De Baets, B., De Meyer, H.: Transitivity frameworks for reciprocal relations: cycle transitivity versus FG-transitivity. Fuzzy Sets Syst. **152**, 249–270 (2005)
14. Dubois, D.: The role of fuzzy sets in decision sciences: Old techniques and new directions. Fuzzy Sets Syst. **184**, 3–28 (2011)
15. McClellon, M.: Confidence models of incomplete preferences. Math. Soc. Sci. **83**, 30–34 (2016)
16. Chateauneuf, A., Faro, J.H.: On the confidence preferences model. Fuzzy Sets Syst. **188**(1), 1–15 (2012)
17. Aliev, R.A.: Uncertain Computation-Based Decision Theory, p. 540. World Scientific Publishing Co Pte Ltd. (2017)
18. Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Izadikhah, M.: An algorithmic method to extend TOPSIS for decision-making problems with interval data. Appl. Math. Comput. **175**, 1375–1384 (2006) AQ4

Author Query Form

Book ID :	**466534_1_En**
Chapter No :	**108**

Please ensure you fill out your response to the queries raised below and return this form along with your corrections.

Dear Author,
During the process of typesetting your chapter, the following queries have arisen. Please check your typeset proof carefully against the queries listed below and mark the necessary changes either directly on the proof/online grid or in the 'Author's response' area provided below

Query Refs.	Details Required	Author's Response
AQ1	The closing parenthesis does not have a corresponding opening parenthesis in table caption 4. Please insert the parenthesis in the appropriate position.	
AQ2	Please check and confirm if the inserted citations of Tables 3–5 are correct. If not, please suggest an alternate citations. And also check tables have been renumbered to maintain sequential order.	
AQ3	To maintain sequential order, Tables and their citations have been renumbered. Please check and correct if necessary.	
AQ4	Reference [18] is given in the list but not cited in the text. Please cite this in text or delete this from the list.	

MARKED PROOF

Please correct and return this set

Please use the proof correction marks shown below for all alterations and corrections. If you wish to return your proof by fax you should ensure that all amendments are written clearly in dark ink and are made well within the page margins.

Instruction to printer	*Textual mark*	*Marginal mark*
Leave unchanged	• • • under matter to remain	
Insert in text the matter indicated in the margin		New matter followed by or
Delete	/ through single character, rule or underline or through all characters to be deleted	or
Substitute character or substitute part of one or more word(s)	/ through letter or through characters	new character / or new characters /
Change to italics	under matter to be changed	
Change to capitals	under matter to be changed	
Change to small capitals	under matter to be changed	
Change to bold type	under matter to be changed	
Change to bold italic	under matter to be changed	
Change to lower case	Encircle matter to be changed	
Change italic to upright type	(As above)	
Change bold to non-bold type	(As above)	
Insert 'superior' character	/ through character or where required	or under character e.g. or
Insert 'inferior' character	(As above)	over character e.g.
Insert full stop	(As above)	⊙
Insert comma	(As above)	,
Insert single quotation marks	(As above)	or and/or or
Insert double quotation marks	(As above)	or and/or or
Insert hyphen	(As above)	
Start new paragraph		
No new paragraph		
Transpose		
Close up	linking characters	
Insert or substitute space between characters or words	/ through character or where required	
Reduce space between characters or words	between characters or words affected	