Advances in Intelligent Systems and Computing 896

Rafik A. Aliev
Janusz Kacprzyk
Witold Pedrycz
Mo. Jamshidi
Fahreddin M. Sadikoglu *Editors*

13th International Conference on Theory and Application of Fuzzy Systems and Soft Computing — ICAFS-2018

Springer

Advances in Intelligent Systems and Computing

Volume 896

Series editor

Janusz Kacprzyk, Systems Research Institute, Polish Academy of Sciences,
Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

H. Berenji, USA
V. Loia, Italy
S. Ulyanov, Russia
K. Bonfig, Germany
P. Moog, Germany
H. Uzunboylu, North Cyprus
A. Danandeh Mehr, Iran
A. Musayev, Azerbaijan
A. Veliyev, Azerbaijan
D. Dubois, Japan
M. Nikravesh, USA
R. Yager, USA
D. Enke, USA
V. Niskanen, Finland

Organizing Committee

Chairman

U. Eberhardt, Germany

Co-chairmen

T. Abdullayev, Azerbaijan
O. Huseynov, Azerbaijan
E. Tuncel, North Cyprus

Members

L. Gardashova, Azerbaijan
A. Alizadeh, Azerbaijan
S. Uzelaltinbulat, North Cyprus
Gunay Sadikoglu, North Cyprus
B. Guirimov, Azerbaijan
A. Guliyev, Azerbaijan
K. Jabbarova, Azerbaijan
M. M. M. Elamin, North Cyprus
M. A. Salahli, Turkey
Şahin Akdağ, North Cyprus

Conference Organizing Secretariat

Azadlig Ave. 20, AZ 1010 Baku, Azerbaijan
Phone: +99 412 493 45 38, Fax: +99 412 598 45 09
E-mail: raliev@asoa.edu.az

Rafik A. Aliev · Janusz Kacprzyk
Witold Pedrycz · Mo. Jamshidi
Fahreddin M. Sadikoglu
Editors

13th International Conference on Theory and Application of Fuzzy Systems and Soft Computing — ICAFS-2018

Springer

Editors
Rafik A. Aliev
Joint MBA Program, Azerbaijan State Oil and Industry University
Baku, Azerbaijan

Janusz Kacprzyk
Systems Research Institute
Polish Academy of Sciences
Warsaw, Poland

Witold Pedrycz
Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB, Canada

Mo. Jamshidi
Department of Electrical and Computer Engineering
University of Texas at San Antonio
San Antonio, TX, USA

Fahreddin M. Sadikoglu
Department of Mechatronics
Near East University
Nicosia TRNC, Turkey

ISSN 2194-5357 ISSN 2194-5365 (electronic)
Advances in Intelligent Systems and Computing
ISBN 978-3-030-04163-2 ISBN 978-3-030-04164-9 (eBook)
https://doi.org/10.1007/978-3-030-04164-9

Library of Congress Control Number: 2018960997

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Organization

Chairman

R. A. Aliev, Azerbaijan

Co-chairmen and Guest Editors

J. Kacprzyk, Poland
M. Jamshidi, USA
W. Pedrycz, Canada
F. S. Sadikoglu, North Cyprus

International Program Committee

R. Abiyev, North Cyprus
B. Fazlollahi, USA
V. Nourani, Iran
I. G. Akperov, Russia
T. Fukuda, Japan
V. Novak, Czech Republic
R. R. Aliev, North Cyprus
M. Gupta, Canada
I. Perfilieva, Czech Republic
F. Aminzadeh, USA
R. Gurbanov, Azerbaijan
H. Prade, France
K. Atanassov, Bulgary
O. Huseynov, Azerbaijan
H. Roth, Germany
N. Yusupbekov, Uzbekistan
Q. Imanov, Azerbaijan
F. Sadikoglu, North Cyprus
A. Averkin, Russia
C. Kahraman, Turkey
M. Salukvadze, Georgia
E. Babaei, Iran
O. Kaynak, Turkey
T. Takagi, Japan
M. Babanli, Azerbaijan
V. Kreinovich, USA
K. Takahashi, Japan
I. Batyrshin, Mexico
D. Kumar Jana, India
V. B. Tarasov, Russia

H. Berenji, USA
V. Loia, Italy
S. Ulyanov, Russia
K. Bonfig, Germany
P. Moog, Germany
H. Uzunboylu, North Cyprus
A. Danandeh Mehr, Iran
A. Musayev, Azerbaijan
A. Veliyev, Azerbaijan
D. Dubois, Japan
M. Nikravesh, USA
R. Yager, USA
D. Enke, USA
V. Niskanen, Finland

Organizing Committee

Chairman

U. Eberhardt, Germany

Co-chairmen

T. Abdullayev, Azerbaijan
O. Huseynov, Azerbaijan
E. Tuncel, North Cyprus

Members

L. Gardashova, Azerbaijan
A. Alizadeh, Azerbaijan
S. Uzelaltinbulat, North Cyprus
Gunay Sadikoglu, North Cyprus
B. Guirimov, Azerbaijan
A. Guliyev, Azerbaijan
K. Jabbarova, Azerbaijan
M. M. M. Elamin, North Cyprus
M. A. Salahli, Turkey
Şahin Akdağ, North Cyprus

Conference Organizing Secretariat

Azadlig Ave. 20, AZ 1010 Baku, Azerbaijan
Phone: +99 412 493 45 38, Fax: +99 412 598 45 09
E-mail: raliev@asoa.edu.az

Contents

Weighted Estimate of Country Risk Using a Fuzzy Method of Maxmin Convolution

Sevinj Babayeva[1(✉)], Inara Rzayeva[2], and Tofig Babayev[1]

[1] Institute of Control Systems of ANAS, Baku AZ1141, Azerbaijan
babayevasevinj@yahoo.com
[2] Azerbaijan State University of Economics, Baku AZ1101, Azerbaijan

Abstract. Weighted attribute estimates and fuzzy method of maximin convolution based two approaches to evaluation the levels of country risk are considered. To obtain the final estimates of the country risk levels for an arbitrary set of alternatives these approaches are used on the base of expert conclusions regarding factors of country risk. The study is completed by comparative analysis of finale estimates of country risks.

Keywords: Country risk · Concordance coefficient · Fuzzy set

1 Introduction

Along with force majeure situations, country risks carry the dangers of political, legal, and socio-economic character. Therefore, to guarantee protection against such threats, it is necessary to take into account the economic and political situation in the aggregate (especially in emerging markets), which, in fact, predetermined the introduction of the concept of "country risk". Country risk (CR) is a multi-factor category characterized by a combined system of financial and economic, socio-political, and legal factors that distinguish the market of any country [1]. According to the degree of CR the countries are ranked by quantitative assessments. This ranking includes the following stages: (1) selection of financial and economic, socio-political and legal variables of CR; (2) identification of the weights of the selected variables of CR on the base of their relative influence on the CR-level; (3) expert estimation of CR factors using the established scale; (4) determination of weighted index reflecting the CR-level.

At present, many world rating agencies, and international institutions, such as Euromoney, Institutional Investor, Mood's Investor Service, the European Bank for Reconstruction and Development (EBRD), the World Bank (WB), etc., are currently ranking countries according to their CR-level. At the same time, existing approaches are conditioned by qualitative and/or quantitative, economic, combined, and structurally qualitative methods for estimating of CR.

R. A. Aliev et al. (Eds.): ICAFS-2018, AISC 896, pp. 559–567, 2019.
https://doi.org/10.1007/978-3-030-04164-9_74

2 Problem Definition

Well-known auditing firm Pricewaterhouse Coopers uses a limited set of variables to formulate the ratings of the investment attractiveness of states. These variables are formulated and denoted in the following form: x_1 – the level of corruption; x_2 – compliance of legislation; x_3 – the level of economic growth; x_4 – state policy on accounting and control; x_5 – state regulation [1]. On the base of above list of variables for the CR-aggregation it is necessary to conduct a preliminary expert analysis by conducting the comparative qualitative assessment of the risk factors (by simple ranking method on the base of expert preferences) and quantitative estimation of the weights of these factors (by applying the normalized values of the weights). Further, by determining the degree of consistency of the expert estimates relative to the priority x_i ($i = 1 \div 5$) and their generalized weights it is necessary to compile the total index in the range from 0% to 100%.

Assuming the variables x_i ($i = 1 \div 5$) as qualitative characteristics that exert relative effects on the CR-level, in addition to the above, it is necessary to carry out a multi-criteria evaluation of the alternative (hypothetical countries) relative to their SR-levels by a fuzzy maxmin convolution method.

3 Ranking of CR-Factors in the Orders of Experts' Preferences

Suppose that expert evaluations of the degrees of importance of CR-factors x_i ($i = 1 \div 5$) are determined by independent questionnaire of 15 profile specialists. Each expert was asked to arrange the variable x_i according to the principle: the most important variable should be designated by the number "1", followed the less important one by the number "2", and further in descending order of importance. The rank estimates obtained in this way are summarized in Table 1.

Table 1. Ranking of CR-factors in the orders of experts' preferences.

Expert	Estimated factors and their ranks (r_{ij})					Expert	Estimated factors and their ranks (r_{ij})				
	x_1	x_2	x_3	x_4	x_5		x_1	x_2	x_3	x_4	x_5
01	1	2	4	3	5	09	1	3	2	4	5
02	1	3	2	4	5	10	1	3	2	5	4
03	2	1	5	4	3	11	1	3	4	2	5
04	1	2	4	5	3	12	1	2	3	5	4
05	2	1	3	4	5	13	2	1	4	3	5
06	1	2	4	3	5	14	3	1	2	4	5
07	2	1	4	3	5	15	1	2	5	4	3
08	1	2	4	5	3	$\sum r_{ij}$	21	29	52	55	65

To determine the degree of consistency of expert opinions, the Kendall concordance coefficient is applied, which demonstrates a multiple rank correlation of expert opinions. According to [2, 3], this coefficient is calculated by the formula:

$$W = \frac{12 \cdot S}{m^2(n^3 - n)}, \quad (1)$$

where m is the number of experts; n is the number of CR-factor; S is the deviation of expert conclusions from the average value of the ranking of the CR-factor, which is calculated, for example, by the formula (3):

$$S = \sum_{i=1}^{n} \sum_{j=1}^{m} [r_{ij} - m(n+1)/2]^2, \quad (2)$$

where $r_{ij} \in \{1, 2, 3, 4, 5\}$ is the rank of the i-th CR-factor, established by the j-th expert. In the case under consideration (see Table 1) the value of S is 1450 and Kendall concordance coefficient is $W = 12 \cdot 1450/[15^2(5^3 - 5)] = 0.6444$. Condition $W > 0.6$ testifies the *strong* consistency of expert opinions on the importance of CR-factors.

4 Weight Identification of CR-Factors

Now, suppose that at the preliminary stage of the independent questionnaire, each expert was also instructed to establish the values of the normalized estimates of CR-factors weights. The results of this questionnaire are summarized in Table 2.

Table 2. The values of the normalized estimates of CR-factors weights.

Expert	Normalized weights for factors (α_{ij})					Expert	Normalized weights for factors (α_{ij})				
	x_1	x_2	x_3	x_4	x_5		x_1	x_2	x_3	x_4	x_5
01	0.300	0.250	0.150	0.225	0.075	09	0.275	0.175	0.200	0.100	0.250
02	0.350	0.175	0.200	0.150	0.125	10	0.300	0.200	0.250	0.100	0.150
03	0.225	0.250	0.150	0.175	0.200	11	0.300	0.175	0.150	0.250	0.125
04	0.275	0.250	0.175	0.100	0.200	12	0.300	0.250	0.200	0.100	0.150
05	0.250	0.275	0.200	0.175	0.100	13	0.225	0.250	0.175	0.200	0.150
06	0.300	0.250	0.150	0.200	0.100	14	0.200	0.300	0.250	0.150	0.100
07	0.200	0.375	0.150	0.175	0.100	15	0.300	0.250	0.125	0.150	0.175
08	0.325	0.300	0.150	0.025	0.200	$\sum r_{ij}$	4.125	3.725	2.675	2.275	2.200

Starting from the data of Table 2 let us make preliminary calculations for the subsequent identification of the CR-factors weights. It is necessary to define the group estimates of the CR-factors and the numerical characteristics (degrees) of competence of all experts. To calculate the average value α_i from the i-th group of normalized estimates of the CR-factors weights, let us use the weighted degrees of expert competence by following difference equation:

$$\alpha_i(t+1) = \sum_{j=1}^{m} w_j(t)\alpha_{ij}, \tag{3}$$

where $w_j(t)$ is the weight characterizing the level of competency of the j-th expert ($j = 1 \div m$) at time t. In this case, the process of finding group estimates of the normalized values has an iterative character, which is completed under following condition:

$$\max_i\{|\alpha_i(t+1) - \alpha_i(t)|\} \le \varepsilon \tag{4}$$

where ε is the permissible accuracy of calculations, which is set in advance. In this case, let it be ε = 0.0001.

Let at initial stage $t = 0$ experts have the same levels of competence. Then, assuming for the general case $w_j(0) = 1/m$ as the initial value of the level of competence of the j-th expert, for the i-th group of normalized estimates of the CR-factors weights the average value in the first approximation is obtained from the partial equality:

$$\alpha_i(1) = \sum_{j=1}^{m} w_j(0)\alpha_{ij} = \frac{1}{m}\sum_{j=1}^{m} \alpha_{ij} \tag{5}$$

In accordance with (5), the averaged estimates of the CR-factors weights by groups in the first approximation are the following corresponding numbers: $\{\alpha_1(1); \alpha_2(1); \alpha_3(1); \alpha_4(1); \alpha_5(1)\}$ = {0.27500; 0.24833; 0.17833; 0.15167; 0.14667}. It is not difficult to see that requirement (4) is not satisfied for the first approximation. Therefore, in order to proceed to the next stage, let us calculate the rating coefficient as: $\eta(1) = \sum_{i=1}^{5}\sum_{j=1}^{15} \alpha_i(1)\alpha_{ij}$ =3.2042. Then, according to the following equalities:

$$\begin{cases} w_j(1) = \dfrac{1}{\eta(1)}\displaystyle\sum_{i=1}^{5} \alpha_i(1) \cdot \alpha_{ij}\,(j = \overline{1,\,14}), \\ w_{15}(1) = 1 - \displaystyle\sum_{j=1}^{14} w_j(1),\ \sum_{j=1}^{15} w_j(1) = 1, \end{cases} \tag{6}$$

where $w_{15}(1)$ is the competency indicator of the 15-th expert, let us calculate the of expert competence indicators in the first approximation as: $\{w_1(1); w_2(1); w_3(1); w_4(1); w_5(1); w_6(1); w_7(1); w_8(1); w_9(1); w_{10}(1); w_{11}(1); w_{12}(1); w_{13}(1); w_{14}(1); w_{15}(1)\}$ = {0.0676; 0.0676; 0.0645; 0.0666; 0.0668; 0.0675; 0.0674; 0.0698; 0.0645; 0.0668; 0.0652; 0.0679; 0.0648; 0.0660; 0.0672}.

Now let us compute the average group estimate of the CR-factors in the second approximation by the formula (3), or more precisely from its particular expression: $\alpha_i(2) = \sum_{j=1}^{15} w_j(1)\alpha_{ij}$. In this case, the average estimates of the CR-factors for groups $i = 1 \div 5$ in the second approximation are the numbers: $\{\alpha_1(2); \alpha_2(2); \alpha_3(2); \alpha_4(2); \alpha_5(2)\} = \{0.27547; 0.24876; 0.17821; 0.15116; 0.14640\}$. Checking the obtained values for condition (4) and convincing that it is not satisfied again: $\max\{|\alpha_i(2) - {}_i(1)|\} = \max\{|0.2755 - 0.2750|; |0.2488 - 0.2483|; |0.1782 - 0.1783|; |0.1512 - 0.1517|; |0.1464 - 0.1467|\} = 0.0005 > \varepsilon$, it is necessary to calculate the rating coefficient as: $\eta(2) = \sum_{i=1}^{5}\sum_{j=1}^{15} \alpha_i(2)\alpha_{ij} = 3.2056$. Then the indicators of expert competence at the second approximation $w_j(2)$ $(j = 1 \div 15)$ will be following numbers: $\{w_1(2); w_2(2); w_3(2); w_4(2); w_5(2); w_6(2); w_7(2); w_8(2); w_9(2); w_{10}(2); w_{11}(2); w_{12}(2); w_{13}(2); w_{14}(2); w_{15}(2)\} = \{0.0676; 0.0676; 0.0645; 0.0666; 0.0668; 0.0675; 0.0674; 0.0699; 0.0645; 0.0668; 0.0652; 0.0679; 0.0647; 0.0660; 0.0672\}$.

The average group estimates of the CR-factors in the third approximation are obtained from the following particular expression of formula (3), namely: $\alpha_i(3) = \sum_{j=1}^{15} w_j(2)\alpha_{ij}$. In this case, the average values of the CR-factors for the groups $i = 1 \div 5$ in the third approximation are the following numbers: $\{\alpha_1(3); \alpha_2(3); \alpha_3(3); \alpha_4(3); \alpha_5(3)\} = \{0.27547; 0.24876; 0.17821; 0.15115; 0.14640\}$. The accuracy of the group estimates x_i $(i = 1 \div 5)$ in the third approximation already satisfies the condition (4), that is, $\max\{|\alpha_i(3) - \alpha_i(2)|\} = \max\{|0.27547 - 0.27547|; |0.24876 - 0.24876|; |0.17821 - 0.17821|; |0.15115 - 0.15116|; |0.1464 - 0.1464|\} = 0.00001 < \varepsilon$, which is the basis for stopping calculations. Then $\{\alpha_1(3); \alpha_2(3); \alpha_3(3); \alpha_4(3); \alpha_5(3)\}$ are the summarized weights of CR-factors x_i $(i = 1 \div 5)$.

5 Determination of the Weighted CR-Level

The method of expert evaluations supposes discussing the factors that influence to the CR-level by the group of especially involved specialists. Each of them is given a list of possible risks on the basis of variables x_i $(i = 1 \div 5)$ and is offered to estimate of the probability of their occurrence in percentage terms on the base of the following five-point rating system: 5 – INSIGNIFICANT RISK; 4 – MOST PROBABLY THE RISK SITUATION DO NOT OCCUR; 3 – ABOUT THE POSSIBILITY OF RISK IT IS IMPOSSIBLE TO SAY ANYTHING DEFINITELY; 2 – THE RISK SITUATION WILL MOST PROBABLY COME; 1 – THE RISK SITUATION WILL MOST CERTAINLY COME. Further, expert judgements are analyzed for consistency by the rule: the maximum permissible difference between two expert opinions for any kind of risk with respect to x_i $(i = 1 \div 5)$ should not exceed 3. This rule allows filter inadmissible deviations in expert judgements of the probability of occurrence of the risk for each CR-factor. The summary index, theoretically ranging from 0 to 100 can be calculate by following assessment criterion:

$$R = \frac{\sum_{i=1}^{5} \alpha_i e_i}{\max_i \sum_{i=1}^{5} \alpha_i e_i} \times 100, \quad (7)$$

where α_i is the weight of the importance of the i-th CR-factor; e_i is the five-point evaluation system based expert judgement of the risk probability for i-th CR-factor. The minimum index symbolizes the maximum risk, and vice versa. CR-level is established on the base of the graduation of the resulting weighted estimates.

Suppose that the expert community is offered to test 10 alternative countries a_k ($k = 1 \div 10$) by the five-point system: every expert need to assess the degree of influence of financial and economic, socio-political, and state-legal factors in these countries on their CR-level. So, estimates of the CR levels of these countries are obtained on the base of consolidated (averaged) expert opinions and application of the assessment criterion (7). Obtained estimates are summarized in the form of Table 3.

Table 3. Indexes of the CR-levels for alternative countries.

Alternative countries	Weights of CR-factors					Index
	α_1	α_2	α_3	α_4	α_5	
a_1	4.50	4.75	4.5	4.75	4.25	91.27
a_2	4.85	4.50	4.55	2.75	3.75	84.62
a_3	3.75	4.00	3.25	3.85	3.25	73.30
a_4	4.25	3.45	2.85	2.75	1.85	64.47
a_5	4.00	2.55	3.00	2.25	1.85	57.64
a_6	3.55	2.85	2.00	1.25	0.85	47.13
a_7	2.25	1.75	1.25	1.85	1.50	35.54
a_8	2.25	1.85	1.25	0.75	0.25	29.06
a_9	5.00	4.75	4.85	4.85	4.75	97.04
a_{10}	3.25	2.85	3.75	4.25	3.50	68.55

6 Ranking CR-Levels of the Countries Using the Fuzzy Method of Maxmin Convolution

The processing of expert judgements by the five-point system presented in Table 3 concerning the CR-factors for alternative a_k ($k = 1 \div 10$) one can be carried out using the mathematical apparatus of the fuzzy sets theory by three stages.

Step 1. Construction of the membership function (fuzzification), which appropriates to the evaluation concept "NON-EXISTING RISK" [4]. In the case under consideration, this term can be reflected in the form of a fuzzy subset of the discrete finite set of estimated alternatives (in our case, countries) $\{a_1, a_2, ..., a_{10}\}$ in the following form: $A_i = \{\mu_{Ai}(a_1)/a_1; \ldots; \mu_{Ai}(a_{10})/a_{10}\}$, where $\mu_{Ai}(a_t)$ ($t = 1 \div 10$) is the value of the membership function of the fuzzy set A_i, which determines the ratio of the t-th country to the evaluation criterion A_i = NON-EXISTING RISK. As the membership function it is possible to choose a Gaussian function of the form: $\mu_{Ai}(a_t) = exp\{-[e_i(a_t) - 5]^2/\sigma_i^2\}$, where $e_i(a_t)$ is the consolidated expert judgement for the country a_t ($t = 1 \div 10$) obtained by five-point scale for compliance with the risk of the i-th factor as non-existent; σ_i^2 is the density of the location of the nearest elements, which is chosen as equal to 4 for all cases of the fuzzification [5].

Step 2. Determination of concrete values of the membership function $\mu_{Ai}(a_t)$ ($t = 1 \div 10$) according to the criteria A_i. In this case, assumed that x_i ($i = 1 \div 5$) are linguistic variables, it is possible to represent one of their terms, namely: "NON-EXISTING RISK" by fuzzy subset A_i of the discrete universe $U = \{a_1, a_2, ..., a_{10}\}$ as follows [4, 5]:

- $A_1 = \{0.9394/a_1; 0.9944/a_2; 0.6766/a_3; 0.8688/a_4; 0.7788/a_5; 0.5912/a_6; 0.1510/a_7; 0.1510/a_8; 1/a_9; 0.4650/a_{10}\}$;
- $A_2 = \{0.9845/a_1; 0.9394/a_2; 0.7788/a_3; 0.5485/a_4; 0.2230/a_5; 0.3149/a_6; 0.0713/a_7; 0.0837/a_8; 0.9845/a_9; 0.3149/a_{10}\}$;
- $A_3 = \{0.9394/a_1; 0.9506/a_2; 0.4650/a_3; 0.3149/a_4; 0.3679/a_5; 0.1054/a_6; 0.0297/a_7; 0.0297/a_8; 0.9944/a_9; 0.6766/a_{10}\}$;
- $A_4 = \{0.9845/a_1; 0.2821/a_2; 0.7185/a_3; 0.2821/a_4; 0.1510/a_5; 0.0297/a_6; 0.0837/a_7; 0.0109/a_8; 0.9944/a_9; 0.8688/a_{10}\}$;
- $A_5 = \{0.8688/a_1; 0.6766/a_2; 0.4650/a_3; 0.0837/a_4; 0.0837/a_5; 0.0135/a_6; 0.0468/a_7; 0.0036/a_8; 0.9845/a_9; 0.5698/a_{10}\}$.

Step 3. To identify the best alternative the convolution of available information. The set of optimal alternatives A is determined by intersection of fuzzy sets containing estimates of alternatives according to the NON-EXISTING RISK criterion [4]. In this case, the rule for choosing the best alternative is

$$A = A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5. \tag{8}$$

Having the maximum value of the membership function of the fuzzy set A alternative is considered optimal. According to [5], the intersection of fuzzy sets appropriates to the choice of the minimum value for the alternative a_t ($t = 1 \div 10$) is

$$\mu_A(a_t) = \min_i\{\mu_{A_i}(a_t)\}. \tag{9}$$

According to (8) and (9) the set of optimal alternatives is formed as follows [5]:

A = {min{0.9394; 0.9845; 0.9394; 0.9845; 0.8688}, min{0.9944; 0.9394; 0.9506; 0.2821; 0.6766}, min{0.6766; 0.7788; 0.4650; 0.7185; 0.4650}, min{0.8688; 0.5485; 0.3149; 0.2821; 0.0837}, min{0.7788; 0.2230; 0.3679; 0.1510; 0.0837}, min{0.5912; 0.3149; 0.1054; 0.0297; 0.0135}, min{0.1510; 0.0713; 0.0297; 0.0837; 0.0468}, min {0.1510; 0.0837; 0.0297; 0.0109; 0.0036}, min{1.0000; 0.9845; 0.9944; 0.9944; 0.9845}, min{0.4650; 0.3149; 0.6766; 0.8688; 0.5698}}.

The resulting priority vector of alternatives is $\max_t\{\mu_A(a_t)\}$ = max{0.8688; 0.2821; 0.4650; 0.0837; 0.0135; 0.0297; 0.0036; 0.9845; 0.3149}.

Thus, from the point of view of the CR-level the best alternative is the country a_9, which corresponds to the value of 0.9845. Next in descending order: $a_1 \rightarrow 0.8688$,

$a_3 \to 0.4650$, $a_{10} \to 0.3149$, $a_2 \to 0.2821$, $a_4 \to 0.0837$, $a_5 \to 0.0837$, $a_7 \to 0.0297$, $a_6 \to 0.0135$, $a_8 \to 0.0036$.

Table 4. The comparison of summarized results of CR-levels estimating.

Alternative countries	Weight-counting technique		Maxmin convolution method	
	Summarized estimate	Order	Summarized estimate	Order
a_1	91.27	2	0.8688	2
a_2	84.62	3	0.2821	5
a_3	73.30	4	0.4650	3
a_4	64.47	6	0.0837	6
a_5	57.64	7	0.0837	7
a_6	47.13	8	0.0135	9
a_7	35.54	9	0.0297	8
a_8	29.06	10	0.0036	10
a_9	97.04	1	0.9845	1
a_{10}	68.55	5	0.3149	4

7 Conclusion

Within the framework of the first approach the generalized values of weights x_i ($i = 1 \div 5$) are established on the base of the agreed expert judgements on the priority of the CR-factors. It becomes the basis for the reasoned formation of the final estimates of the CR-levels according to the established comparison test at the scale of the segment [0; 100]. The fuzzy maxmin convolution method, which is the essence of the second approach, solves the problem by using another way of aggregating of expert judgements of the CR-factors. A comparison of summarized results of CR-levels estimating of hypothetical alternatives (countries) a_t ($t = 1 \div 10$) obtained by both methods is presented in Table 4, which shows that the orders of some estimates of the CR-levels do not coincide.

Acknowledgement. The authors consider necessary to express their sincere appreciation to Professor R.R. Rzayev for the help that he rendered during the process of writing and preparing this article.

References

1. PwC Global Risk podcast series: https://www.pwc.com/gx/en/services/advisory/consulting/risk/cosoerm-framework/podcasts.html. Accessed 15 May 2018
2. Lin, A.S.: A note on the concordance correlation coefficient. Biometrics **56**, 324–325 (2012)
3. Lin, A.S., Wu, W.: Statistical Tools for Measuring Agreement. Springer, New York (2012). https://doi.org/10.1007/978-1-4614-0562-7

4. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning. Inf. Sci. **8**(3), 199–249 (1965)
5. Rzayev, R.R.: Analytical Support for Decision-Making in Organizational Systems. Palmerium Academic Publishing, Saarbruchen (2016). (in Russian)