

Advances in Intelligent Systems and Computing 896

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Fahreddin M. Sadikoglu *Editors*

13th International Conference on Theory and Application of Fuzzy Systems and Soft Computing — ICAFS-2018

Advances in Intelligent Systems and Computing

Volume 896

Series editor

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ISSN 2194-5357 ISSN 2194-5365 (electronic)
Advances in Intelligent Systems and Computing
ISBN 978-3-030-04163-2 ISBN 978-3-030-04164-9 (eBook)
<https://doi.org/10.1007/978-3-030-04164-9>

Library of Congress Control Number: 2018960997

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Contents

From Status Quo Bias to Innovative Multiagent Decisions Under Fuzzy Preferences and Fuzzy Majority	1
Janusz Kacprzyk	
Bimodal Information Clustering Methods	2
R. A. Aliev	
Maximum Likelihood Estimation from Interval-Valued Data. Application to Fuzzy Clustering	3
Hani Hamdan	
Theory and Practice of Material Development Under Imperfect Information	4
M. B. Babanli	
Forming and Quantifying Consensus in Distributed System Modeling and Group Decision-Making: A Perspective of Granular Computing . . .	15
Witold Pedrycz	
Why Multidimensional Fuzzy Arithmetic?	16
Andrzej Piegat and Marek Landowski	
On Using Fuzzy Sets in Healthcare Process Analysis	24
Uzay Kaymak	
Optimization of Jobs in GIS by Coloring of Fuzzy Temporal Graph. . .	25
Alexander Bozhenyuk, Stanislav Belyakov, and Janusz Kacprzyk	
Algebraic Properties of Z-Numbers Under Multiplicative Arithmetic Operations	33
R. A. Aliev and A. V. Alizadeh	
Z-Number Based TOPSIS Method in Multi-Criteria Decision Making	42
Latafat A. Gardashova	

Comparative Analysis of Artificial Intelligence Based Methods for Prediction of Precipitation. Case Study: North Cyprus	51
Selin Uzelaltinbulat, Fahreddin Sadikoglu, and Vahid Nourani	
Analysis and Processing of Information in Economic Problems. Crisp and Fuzzy Technologies	65
Araz R. Aliev, Vagif M. Mamedov, and Gasim G. Gasimov	
Labeled Fuzzy Rough Sets in Multiple-Criteria Decision-Making	73
Alicja Mieszkowicz-Rolka and Leszek Rolka	
Comparing Image Distortion of LSB	82
Yucel Inan	
An Effective Fuzzy Controlled Filter for Feature Extraction Method. . .	91
Mohamad Alshahadat, Bülent Bilgehan, and Cemal Kavalcıoğlu	
Analysis of Prediction Models for Wind Power Density, Case Study: Ercan Area, Northern Cyprus	99
Youssef Kassem, Hüseyin Gökçekuş, and Hüseyin Çamur	
Evaluation of Image Representations for Player Detection in Field Sports Using Convolutional Neural Networks	107
Melike Şah and Cem Direkoğlu	
A Fuzzy Based Gaussian Weighted Moving Windowing for Denoising Electrocardiogram (ECG) Signals	116
Cemal Kavalcıoğlu and Bülent Bilgehan	
Integrated Deep Learning Structures for Hand Gesture Recognition. . .	129
Senol Korkmaz	
Rule Based Intelligent Diabetes Diagnosis System	137
Elbrus Imanov, Hamit Altıparmak, and Gunay E. Imanova	
Intensive Investigation in Differential Diagnosis of Erythematous-Squamous Diseases	146
Idoko John Bush, Murat Arslan, and Rahib Abiyev	
A Computational-Intelligence Based Approach to Diagnosis of Diabetes Mellitus Disease	154
Elif Dogu and Y. Esra Albayrak	
Fuzzy Expert System for Rectal Cancer	160
Yusif R. Aliyarov, Latafat A. Gardashova, and Shamil A. Ahmadov	
Dynamics and Control of HIV/AIDS in Cyprus Using Real Data	167
Evren Hincal, Tamer Sanlidag, Farouk Tijjani Saad, Kaya Suer, Isa Abdullahi Baba, Murat Sayan, Bilgen Kaymakamzade, and Nazife Sultanoglu	

Intuitionistic Fuzzy Sets for Estimating the Parameters of Distributive Task	178
Alexander Bozhenyuk, Margarita Knyazeva, and Olesiya Kosenko	
Fuzzy Logic Based Modelling of Decision Buying Process	185
Gunay Sadikoglu and Tulen Saner	
An Integrated Fuzzy Decision Framework for Neuromarketing Technology Selection Problem	195
Mehtap Dursun and Nazli Goker	
Performance Indicators Evaluation of Business Process Outsourcing Employing Fuzzy Cognitive Map	201
Nazli Goker, Y. Esra Albayrak, and Mehtap Dursun	
Evaluating Lung Cancer Treatment Techniques Using Fuzzy PROMETHEE Approach	209
Mordecai Maisaini, Berna Uzun, Ilker Ozsahin, and Dilber Uzun	
A Comparative Study on the Economic Development Level of the Countries by Fuzzy DEA Methodologies	216
Mujde Erol Genevois and Michele Cedolin	
Fuzzy Analysis of Macroeconomic Stability	223
Gorkhmaz Imanov, Yadulla Hasanli, and Malahat Murtuzaeva	
Wind Speed Prediction of Four Regions in Northern Cyprus Prediction Using ARIMA and Artificial Neural Networks Models: A Comparison Study	230
Youssef Kassem, Hüseyin Gökçekuş, and Hüseyin Çamur	
Head Movement Mouse Control Using Convolutional Neural Network for People with Disabilities	239
Murat Arslan, Idoko John Bush, and Rahib H. Abiyev	
Data Coding and Neural Network Arbitration for Feasibility Prediction of Car Marketing	249
Adnan Khashman and Gunay Sadikoglu	
Signature Recognition Using Backpropagation Neural Network	256
Yucel Inan and Boran Sekeroglu	
Topological Ordering on Interval Type-2 Fuzzy Graph	262
Margarita Knyazeva, Stanislav Belyakov, and Janusz Kacprzyk	
Z-Number Clustering Based on General Type-2 Fuzzy Sets	270
Rafik Aliev and Babek Guirimov	
One Approach to Multi-criteria Evaluation of Alternatives in the Logical Basis of Neural Networks	279
Misir Mardanov and Ramin Rzayev	

Self-confidence Preference Based Decision Making in Personnel Selection	288
S. Z. Eyupoglu and G. E. Imanova	
The Incrementality Issue in the Wu-Mendel Approach for Linguistic Summarization Using IF-THEN Rules	293
Vuqar E. Mirzakhanov and Latafat A. Gardashova	
A Genetic Programming Approach to Forecast Daily Electricity Demand	301
Ali Danandeh Mehr, Farzaneh Bagheri, and Rifat Reşatoğlu	
Teaching Fuzzy Control Using an Embedded Processor	309
Dogan Ibrahim	
PSO Algorithm Applied to Enhance Power Quality of Multilevel Inverter	315
Yahya Naderi, Fahreddin Sadikoglu, and Seyed Hossein Hosseini	
Development of Instruments of Fuzzy Identification of Extended Objects Based on the Results of Satellite Monitoring	325
Imran G. Akperov and Vladimir V. Khramov	
Optimal Placement of Capacitor Using Particle Swarm Optimization . . .	333
Farzad Mohammadzadeh Shahir, Ebrahim Babaei, and Fahreddin Sadikoglu	
Predicting Fresh Water of Single Slope Solar Still Using a Fuzzy Inference System	344
Lida Ebrahimi Vafaei and Melike Sah	
Evaluation of the Impact of State's Administrative Efforts on Tax Potential Using Sugeno-Type Fuzzy Inference Method	352
Samir Rustamov, Akif Musayev, and Shahzada Madatova	
Analysing the Economic Impacts of the Euro for the Three Largest Economies in the Emu (European Monetary Union) and the Place of the Euro in Global Economics	361
Hüseyin Özdeşer	
Evaluation of Tourism Sector Based on the Internal Environment by Using a Fuzzy Approach	371
İhsan Yüksel, Metin Dağdeviren, and Gülsüm Alicioğlu	
Evaluation of the Impact of the Changing the Term of Tax Liability Performance on Tax Receipts by Minmax Composition Method	378
A. A. Musayeva, T. M. Musayev, and M. Kh. Gazanfarli	

Evaluating US Dollar Index Movements Using Markov Chains-Fuzzy States Approach	386
Berna Uzun and Ersin Kırıl	
The Statistical Reasoning of the Occupational Attitudes of Students. Case Study: Gastronomy and Culinary Arts Program	392
Serdar Oktay and Saide Sadıkoğlu	
Statistical Reasoning of Education Managers Opinions on Institutional Strategic Planning	399
Gülsün Başarı, Ali Aktepebaşı, Ediz Tuncel, Emete Yağcı, and Şahin Akdağ	
The Impact of Using Social Media on University Students Socialization: Statistical Reasoning	404
Şahin Akdağ, Huseyin Bicen, Gulsun Basari, and Ahmet Savasan	
Statistical and Structural Equation Modelling the Relationship Between Creativity and Material Design Self-efficacy Beliefs of Preschool Woman Preservice Teachers	410
Konul Memmedova, Dervise Amca Toklu, and Saide Sadikoglu	
Directions of Complex Regional Socio-Economic Development Based on Cognitive Modeling of Realization of Investment Strategy	417
Imran Akperov, Nataliya Brukhanova, and Dmitriy Dynnik	
Expert-Analytical Support for the Document Marking Process Using a Fuzzy Analysis of Data Confidentiality	426
Ramin Rzayev, Elkhan Aliyev, and Afet Suleymanova	
An Interval Number Based Input-Output Analysis for a Regional Economy	435
Behiye Cavusoglu, Serife Z. Eyupoglu, Pinar Sharghi, and Tulen Saner	
Fuzzy Stability Analysis of an Isothermal First-Order Reaction	444
Mahsati Babanlı	
Prioritization of Mosque Facility Site Selection Criteria Under Fuzzy Environment	451
Abdurrahman Yagmur Toprakli, Mehmet Kabak, Eren Özceylan, and Aylin Adem	
Statistical Computation of the Effect of Using Mobile Applications as a Travel Information Tool	458
Huseyin Bicen and Ahmet Arnavut	
Improvement in Strength of Radio Wave Propagation Outside the Coverage Area of the Mobile Towers for Cellular Mobile WiFi.	464
Jamal Fathi	

Application of Interval Approximation Method of a Fuzzy Number to the Supplier Selection	472
Kamala Aliyeva	
Multifactor Personnel Selection by the Fuzzy TOPSIS Method	478
Kamala Aliyeva	
Fractal Analysis of Chaotic Fluctuations in Oil Production	484
E. E. Ramazanov, A. A. Abbasov, H. Kh. Malikov, and A. A. Suleymanov	
Non-parametric Criteria of Chaotic Data Analysis in Oil Production . . .	491
T. Sh. Salavatov, A. A. Abbasov, H. Kh. Malikov, D. F. Guseynova, and A. A. Suleymanov	
An Application of the VIKOR Method to Decision Making in Investment Problem Under Z-Valued Information	499
Konul Jabbarova and Naila Hasanova	
Energy Consumption Prediction of Residential Buildings Using Fuzzy Neural Networks	507
Sanan Abizada and Esmira Abiyeva	
Classification of Diseases on Chest X-Rays Using Deep Learning	516
Sertan Kaymak, Khaled Almezghwi, and Almaki A. S. Shelag	
The Ecosystem of ICT Based Innovation in the Public Sector. The Role of Public Procurement	524
Ani Matei, Carmen Săvulescu, and Corina-Georgiana Antonovici	
Fuzzy Set-Theory Analysis of Deregulations Effects on the National Taxable Capacity	533
Rovshan Akbarov, Musa Aghamaliyev, and Emin Garibli	
Ranking of Universities on the Base of Fuzzy Evaluation Procedure of Their Index of Competitiveness	542
Zeynal Jamalov, Alla Khudadova, Inara Abasova, and Narmina Aliyeva	
Approach to the Increasing the Validity of the Position-Binary Method for Cyclic Signal Recognition	551
Oktay Nusratov, Asker Almasov, and Hafiz Bayramov	
Weighted Estimate of Country Risk Using a Fuzzy Method of Maxmin Convolution	559
Sevinj Babayeva, Inara Rzayeva, and Tofig Babayev	
Forecasting the Solvency of Legal Entities on the Base of Fuzzy Modeling of the Time Series of Their Financial Indicators	568
Elchin Aliyev, Hajar Shikhaliyeva, Zaur Gaziyeu, and Adila Ali	



Weighted Estimate of Country Risk Using a Fuzzy Method of Maxmin Convolution

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Abstract. Weighted attribute estimates and fuzzy method of maximin convolution based two approaches to evaluation the levels of country risk are considered. To obtain the final estimates of the country risk levels for an arbitrary set of alternatives these approaches are used on the base of expert conclusions regarding factors of country risk. The study is completed by comparative analysis of finale estimates of country risks.

Keywords: Country risk · Concordance coefficient · Fuzzy set

1 Introduction

Along with force majeure situations, country risks carry the dangers of political, legal, and socio-economic character. Therefore, to guarantee protection against such threats, it is necessary to take into account the economic and political situation in the aggregate (especially in emerging markets), which, in fact, predetermined the introduction of the concept of “country risk”. Country risk (CR) is a multi-factor category characterized by a combined system of financial and economic, socio-political, and legal factors that distinguish the market of any country [1]. According to the degree of CR the countries are ranked by quantitative assessments. This ranking includes the following stages: (1) selection of financial and economic, socio-political and legal variables of CR; (2) identification of the weights of the selected variables of CR on the base of their relative influence on the CR-level; (3) expert estimation of CR factors using the established scale; (4) determination of weighted index reflecting the CR-level.

At present, many world rating agencies, and international institutions, such as Euromoney, Institutional Investor, Mood’s Investor Service, the European Bank for Reconstruction and Development (EBRD), the World Bank (WB), etc., are currently ranking countries according to their CR-level. At the same time, existing approaches are conditioned by qualitative and/or quantitative, economic, combined, and structurally qualitative methods for estimating of CR.

2 Problem Definition

Well-known auditing firm Pricewaterhouse Coopers uses a limited set of variables to formulate the ratings of the investment attractiveness of states. These variables are formulated and denoted in the following form: x_1 – the level of corruption; x_2 – compliance of legislation; x_3 – the level of economic growth; x_4 – state policy on accounting and control; x_5 – state regulation [1]. On the base of above list of variables for the CR-aggregation it is necessary to conduct a preliminary expert analysis by conducting the comparative qualitative assessment of the risk factors (by simple ranking method on the base of expert preferences) and quantitative estimation of the weights of these factors (by applying the normalized values of the weights). Further, by determining the degree of consistency of the expert estimates relative to the priority x_i ($i = 1 \div 5$) and their generalized weights it is necessary to compile the total index in the range from 0% to 100%.

Assuming the variables x_i ($i = 1 \div 5$) as qualitative characteristics that exert relative effects on the CR-level, in addition to the above, it is necessary to carry out a multi-criteria evaluation of the alternative (hypothetical countries) relative to their SR-levels by a fuzzy maxmin convolution method.

3 Ranking of CR-Factors in the Orders of Experts' Preferences

Suppose that expert evaluations of the degrees of importance of CR-factors x_i ($i = 1 \div 5$) are determined by independent questionnaire of 15 profile specialists. Each expert was asked to arrange the variable x_i according to the principle: the most important variable should be designated by the number “1”, followed the less important one by the number “2”, and further in descending order of importance. The rank estimates obtained in this way are summarized in Table 1.

Table 1. Ranking of CR-factors in the orders of experts' preferences.

Expert	Estimated factors and their ranks (r_{ij})					Expert	Estimated factors and their ranks (r_{ij})				
	x_1	x_2	x_3	x_4	x_5		x_1	x_2	x_3	x_4	x_5
01	1	2	4	3	5	09	1	3	2	4	5
02	1	3	2	4	5	10	1	3	2	5	4
03	2	1	5	4	3	11	1	3	4	2	5
04	1	2	4	5	3	12	1	2	3	5	4
05	2	1	3	4	5	13	2	1	4	3	5
06	1	2	4	3	5	14	3	1	2	4	5
07	2	1	4	3	5	15	1	2	5	4	3
08	1	2	4	5	3	$\sum r_{ij}$	21	29	52	55	65

To determine the degree of consistency of expert opinions, the Kendall concordance coefficient is applied, which demonstrates a multiple rank correlation of expert opinions. According to [2, 3], this coefficient is calculated by the formula:

$$W = \frac{12 \cdot S}{m^2(n^3 - n)}, \quad (1)$$

where m is the number of experts; n is the number of CR-factor; S is the deviation of expert conclusions from the average value of the ranking of the CR-factor, which is calculated, for example, by the formula (3):

$$S = \sum_{i=1}^n \sum_{j=1}^m [r_{ij} - m(n+1)/2]^2, \quad (2)$$

where $r_{ij} \in \{1, 2, 3, 4, 5\}$ is the rank of the i -th CR-factor, established by the j -th expert. In the case under consideration (see Table 1) the value of S is 1450 and Kendall concordance coefficient is $W = 12 \cdot 1450 / [15^2(5^3 - 5)] = 0.6444$. Condition $W > 0.6$ testifies the *strong* consistency of expert opinions on the importance of CR-factors.

4 Weight Identification of CR-Factors

Now, suppose that at the preliminary stage of the independent questionnaire, each expert was also instructed to establish the values of the normalized estimates of CR-factors weights. The results of this questionnaire are summarized in Table 2.

Table 2. The values of the normalized estimates of CR-factors weights.

Expert	Normalized weights for factors (α_{ij})					Expert	Normalized weights for factors (α_{ij})				
	x_1	x_2	x_3	x_4	x_5		x_1	x_2	x_3	x_4	x_5
01	0.300	0.250	0.150	0.225	0.075	09	0.275	0.175	0.200	0.100	0.250
02	0.350	0.175	0.200	0.150	0.125	10	0.300	0.200	0.250	0.100	0.150
03	0.225	0.250	0.150	0.175	0.200	11	0.300	0.175	0.150	0.250	0.125
04	0.275	0.250	0.175	0.100	0.200	12	0.300	0.250	0.200	0.100	0.150
05	0.250	0.275	0.200	0.175	0.100	13	0.225	0.250	0.175	0.200	0.150
06	0.300	0.250	0.150	0.200	0.100	14	0.200	0.300	0.250	0.150	0.100
07	0.200	0.375	0.150	0.175	0.100	15	0.300	0.250	0.125	0.150	0.175
08	0.325	0.300	0.150	0.025	0.200	$\sum r_{ij}$	4.125	3.725	2.675	2.275	2.200

Starting from the data of Table 2 let us make preliminary calculations for the subsequent identification of the CR-factors weights. It is necessary to define the group estimates of the CR-factors and the numerical characteristics (degrees) of competence of all experts. To calculate the average value α_i from the i -th group of normalized

estimates of the CR-factors weights, let us use the weighted degrees of expert competence by following difference equation:

$$\alpha_i(t+1) = \sum_{j=1}^m w_j(t) \alpha_{ij}, \quad (3)$$

where $w_j(t)$ is the weight characterizing the level of competency of the j -th expert ($j = 1 \div m$) at time t . In this case, the process of finding group estimates of the normalized values has an iterative character, which is completed under following condition:

$$\max_i \{|\alpha_i(t+1) - \alpha_i(t)|\} \leq \varepsilon \quad (4)$$

where ε is the permissible accuracy of calculations, which is set in advance. In this case, let it be $\varepsilon = 0.0001$.

Let at initial stage $t = 0$ experts have the same levels of competence. Then, assuming for the general case $w_j(0) = 1/m$ as the initial value of the level of competence of the j -th expert, for the i -th group of normalized estimates of the CR-factors weights the average value in the first approximation is obtained from the partial equality:

$$\alpha_i(1) = \sum_{j=1}^m w_j(0) \alpha_{ij} = \frac{1}{m} \sum_{j=1}^m \alpha_{ij} \quad (5)$$

In accordance with (5), the averaged estimates of the CR-factors weights by groups in the first approximation are the following corresponding numbers: $\{\alpha_1(1); \alpha_2(1); \alpha_3(1); \alpha_4(1); \alpha_5(1)\} = \{0.27500; 0.24833; 0.17833; 0.15167; 0.14667\}$. It is not difficult to see that requirement (4) is not satisfied for the first approximation. Therefore, in order to proceed to the next stage, let us calculate the rating coefficient as:

$\eta(1) = \sum_{i=1}^5 \sum_{j=1}^{15} \alpha_i(1) \alpha_{ij} = 3.2042$. Then, according to the following equalities:

$$\begin{cases} w_j(1) = \frac{1}{\eta(1)} \sum_{i=1}^5 \alpha_i(1) \cdot \alpha_{ij} \ (j = \overline{1, 14}), \\ w_{15}(1) = 1 - \sum_{j=1}^{14} w_j(1), \sum_{j=1}^{15} w_j(1) = 1, \end{cases} \quad (6)$$

where $w_{15}(1)$ is the competency indicator of the 15-th expert, let us calculate the of expert competence indicators in the first approximation as: $\{w_1(1); w_2(1); w_3(1); w_4(1); w_5(1); w_6(1); w_7(1); w_8(1); w_9(1); w_{10}(1); w_{11}(1); w_{12}(1); w_{13}(1); w_{14}(1); w_{15}(1)\} = \{0.0676; 0.0676; 0.0645; 0.0666; 0.0668; 0.0675; 0.0674; 0.0698; 0.0645; 0.0668; 0.0652; 0.0679; 0.0648; 0.0660; 0.0672\}$.

Now let us compute the average group estimate of the CR-factors in the second approximation by the formula (3), or more precisely from its particular expression:

$\alpha_i(2) = \sum_{j=1}^{15} w_j(1)\alpha_{ij}$. In this case, the average estimates of the CR-factors for groups

$i = 1 \div 5$ in the second approximation are the numbers: $\{\alpha_1(2); \alpha_2(2); \alpha_3(2); \alpha_4(2); \alpha_5(2)\} = \{0.27547; 0.24876; 0.17821; 0.15116; 0.14640\}$. Checking the obtained values for condition (4) and convincing that it is not satisfied again: $\max\{|\alpha_i(2) - \alpha_i(1)|\} = \max\{|0.2755 - 0.2750|; |0.2488 - 0.2483|; |0.1782 - 0.1783|; |0.1512 - 0.1517|; |0.1464 - 0.1467|\} = 0.0005 > \varepsilon$, it is necessary to calculate the rating coefficient as:

$\eta(2) = \sum_{i=1}^5 \sum_{j=1}^{15} \alpha_i(2)\alpha_{ij} = 3.2056$. Then the indicators of expert competence at

the second approximation $w_j(2)$ ($j = 1 \div 15$) will be following numbers: $\{w_1(2); w_2(2); w_3(2); w_4(2); w_5(2); w_6(2); w_7(2); w_8(2); w_9(2); w_{10}(2); w_{11}(2); w_{12}(2); w_{13}(2); w_{14}(2); w_{15}(2)\} = \{0.0676; 0.0676; 0.0645; 0.0666; 0.0668; 0.0675; 0.0674; 0.0699; 0.0645; 0.0668; 0.0652; 0.0679; 0.0647; 0.0660; 0.0672\}$.

The average group estimates of the CR-factors in the third approximation are obtained from the following particular expression of formula (3), namely:

$\alpha_i(3) = \sum_{j=1}^{15} w_j(2)\alpha_{ij}$. In this case, the average values of the CR-factors for the groups

$i = 1 \div 5$ in the third approximation are the following numbers: $\{\alpha_1(3); \alpha_2(3); \alpha_3(3); \alpha_4(3); \alpha_5(3)\} = \{0.27547; 0.24876; 0.17821; 0.15115; 0.14640\}$. The accuracy of the group estimates x_i ($i = 1 \div 5$) in the third approximation already satisfies the condition (4), that is, $\max\{|\alpha_i(3) - \alpha_i(2)|\} = \max\{|0.27547 - 0.27547|; |0.24876 - 0.24876|; |0.17821 - 0.17821|; |0.15115 - 0.15116|; |0.1464 - 0.1464|\} = 0.00001 < \varepsilon$, which is the basis for stopping calculations. Then $\{\alpha_1(3); \alpha_2(3); \alpha_3(3); \alpha_4(3); \alpha_5(3)\}$ are the summarized weights of CR-factors x_i ($i = 1 \div 5$).

5 Determination of the Weighted CR-Level

The method of expert evaluations supposes discussing the factors that influence to the CR-level by the group of especially involved specialists. Each of them is given a list of possible risks on the basis of variables x_i ($i = 1 \div 5$) and is offered to estimate of the probability of their occurrence in percentage terms on the base of the following five-point rating system: 5 – INSIGNIFICANT RISK; 4 – MOST PROBABLY THE RISK SITUATION DO NOT OCCUR; 3 – ABOUT THE POSSIBILITY OF RISK IT IS IMPOSSIBLE TO SAY ANYTHING DEFINITELY; 2 – THE RISK SITUATION WILL MOST PROBABLY COME; 1 – THE RISK SITUATION WILL MOST CERTAINLY COME. Further, expert judgements are analyzed for consistency by the rule: the maximum permissible difference between two expert opinions for any kind of risk with respect to x_i ($i = 1 \div 5$) should not exceed 3. This rule allows filter inadmissible deviations in expert judgements of the probability of occurrence of the risk for each CR-factor. The summary index, theoretically ranging from 0 to 100 can be calculate by following assessment criterion:

$$R = \frac{\sum_{i=1}^5 \alpha_i e_i}{\max_i \sum_{i=1}^5 \alpha_i e_i} \times 100, \quad (7)$$

where α_i is the weight of the importance of the i -th CR-factor; e_i is the five-point evaluation system based expert judgement of the risk probability for i -th CR-factor. The minimum index symbolizes the maximum risk, and vice versa. CR-level is established on the base of the graduation of the resulting weighted estimates.

Suppose that the expert community is offered to test 10 alternative countries a_k ($k = 1 \div 10$) by the five-point system: every expert need to assess the degree of influence of financial and economic, socio-political, and state-legal factors in these countries on their CR-level. So, estimates of the CR levels of these countries are obtained on the base of consolidated (averaged) expert opinions and application of the assessment criterion (7). Obtained estimates are summarized in the form of Table 3.

Table 3. Indexes of the CR-levels for alternative countries.

Alternative countries	Weights of CR-factors					Index
	α_1	α_2	α_3	α_4	α_5	
a_1	4.50	4.75	4.5	4.75	4.25	91.27
a_2	4.85	4.50	4.55	2.75	3.75	84.62
a_3	3.75	4.00	3.25	3.85	3.25	73.30
a_4	4.25	3.45	2.85	2.75	1.85	64.47
a_5	4.00	2.55	3.00	2.25	1.85	57.64
a_6	3.55	2.85	2.00	1.25	0.85	47.13
a_7	2.25	1.75	1.25	1.85	1.50	35.54
a_8	2.25	1.85	1.25	0.75	0.25	29.06
a_9	5.00	4.75	4.85	4.85	4.75	97.04
a_{10}	3.25	2.85	3.75	4.25	3.50	68.55

6 Ranking CR-Levels of the Countries Using the Fuzzy Method of Maxmin Convolution

The processing of expert judgements by the five-point system presented in Table 3 concerning the CR-factors for alternative a_k ($k = 1 \div 10$) one can be carried out using the mathematical apparatus of the fuzzy sets theory by three stages.

Step 1. Construction of the membership function (fuzzification), which appropriates to the evaluation concept “NON-EXISTING RISK” [4]. In the case under consideration, this term can be reflected in the form of a fuzzy subset of the discrete finite set of estimated alternatives (in our case, countries) $\{a_1, a_2, \dots, a_{10}\}$ in the following form: $A_i = \{\mu_{A_i}(a_1)/a_1; \dots; \mu_{A_i}(a_{10})/a_{10}\}$, where $\mu_{A_i}(a_t)$ ($t = 1 \div 10$) is the value of the membership function of the fuzzy set A_i , which determines the ratio of the t -th country to the

evaluation criterion $A_i = \text{NON-EXISTING RISK}$. As the membership function it is possible to choose a Gaussian function of the form: $\mu_{A_i}(a_t) = \exp\{-[e_i(a_t) - 5]^2/\sigma_i^2\}$, where $e_i(a_t)$ is the consolidated expert judgement for the country a_t ($t = 1 \div 10$) obtained by five-point scale for compliance with the risk of the i -th factor as non-existent; σ_i^2 is the density of the location of the nearest elements, which is chosen as equal to 4 for all cases of the fuzzification [5].

Step 2. Determination of concrete values of the membership function $\mu_{A_i}(a_t)$ ($t = 1 \div 10$) according to the criteria A_i . In this case, assumed that x_i ($i = 1 \div 5$) are linguistic variables, it is possible to represent one of their terms, namely: "NON-EXISTING RISK" by fuzzy subset A_i of the discrete universe $U = \{a_1, a_2, \dots, a_{10}\}$ as follows [4, 5]:

- $A_1 = \{0.9394/a_1; 0.9944/a_2; 0.6766/a_3; 0.8688/a_4; 0.7788/a_5; 0.5912/a_6; 0.1510/a_7; 0.1510/a_8; 1/a_9; 0.4650/a_{10}\};$
- $A_2 = \{0.9845/a_1; 0.9394/a_2; 0.7788/a_3; 0.5485/a_4; 0.2230/a_5; 0.3149/a_6; 0.0713/a_7; 0.0837/a_8; 0.9845/a_9; 0.3149/a_{10}\};$
- $A_3 = \{0.9394/a_1; 0.9506/a_2; 0.4650/a_3; 0.3149/a_4; 0.3679/a_5; 0.1054/a_6; 0.0297/a_7; 0.0297/a_8; 0.9944/a_9; 0.6766/a_{10}\};$
- $A_4 = \{0.9845/a_1; 0.2821/a_2; 0.7185/a_3; 0.2821/a_4; 0.1510/a_5; 0.0297/a_6; 0.0837/a_7; 0.0109/a_8; 0.9944/a_9; 0.8688/a_{10}\};$
- $A_5 = \{0.8688/a_1; 0.6766/a_2; 0.4650/a_3; 0.0837/a_4; 0.0837/a_5; 0.0135/a_6; 0.0468/a_7; 0.0036/a_8; 0.9845/a_9; 0.5698/a_{10}\}.$

Step 3. To identify the best alternative the convolution of available information. The set of optimal alternatives A is determined by intersection of fuzzy sets containing estimates of alternatives according to the NON-EXISTING RISK criterion [4]. In this case, the rule for choosing the best alternative is

$$A = A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5. \quad (8)$$

Having the maximum value of the membership function of the fuzzy set A alternative is considered optimal. According to [5], the intersection of fuzzy sets appropiates to the choice of the minimum value for the alternative a_t ($t = 1 \div 10$) is

$$\mu_A(a_t) = \min_i \{\mu_{A_i}(a_t)\}. \quad (9)$$

According to (8) and (9) the set of optimal alternatives is formed as follows [5]:

$A = \{\min\{0.9394; 0.9845; 0.9394; 0.9845; 0.8688\}, \min\{0.9944; 0.9394; 0.9506; 0.2821; 0.6766\}, \min\{0.6766; 0.7788; 0.4650; 0.7185; 0.4650\}, \min\{0.8688; 0.5485; 0.3149; 0.2821; 0.0837\}, \min\{0.7788; 0.2230; 0.3679; 0.1510; 0.0837\}, \min\{0.5912; 0.3149; 0.1054; 0.0297; 0.0135\}, \min\{0.1510; 0.0713; 0.0297; 0.0837; 0.0468\}, \min\{0.1510; 0.0837; 0.0297; 0.0109; 0.0036\}, \min\{1.0000; 0.9845; 0.9944; 0.9944; 0.9845\}, \min\{0.4650; 0.3149; 0.6766; 0.8688; 0.5698\}\}.$

The resulting priority vector of alternatives is $\max_t \{\mu_A(a_t)\} = \max\{0.8688; 0.2821; 0.4650; 0.0837; 0.0135; 0.0297; 0.0036; 0.9845; 0.3149\}.$

Thus, from the point of view of the CR-level the best alternative is the country a_9 , which corresponds to the value of 0.9845. Next in descending order: $a_1 \rightarrow 0.8688$,

$a_3 \rightarrow 0.4650$, $a_{10} \rightarrow 0.3149$, $a_2 \rightarrow 0.2821$, $a_4 \rightarrow 0.0837$, $a_5 \rightarrow 0.0837$, $a_7 \rightarrow 0.0297$,
 $a_6 \rightarrow 0.0135$, $a_8 \rightarrow 0.0036$.

Table 4. The comparison of summarized results of CR-levels estimating.

Alternative countries	Weight-counting technique		Maxmin convolution method	
	Summarized estimate	Order	Summarized estimate	Order
a_1	91.27	2	0.8688	2
a_2	84.62	3	0.2821	5
a_3	73.30	4	0.4650	3
a_4	64.47	6	0.0837	6
a_5	57.64	7	0.0837	7
a_6	47.13	8	0.0135	9
a_7	35.54	9	0.0297	8
a_8	29.06	10	0.0036	10
a_9	97.04	1	0.9845	1
a_{10}	68.55	5	0.3149	4

7 Conclusion

Within the framework of the first approach the generalized values of weights x_i ($i = 1 \div 5$) are established on the base of the agreed expert judgements on the priority of the CR-factors. It becomes the basis for the reasoned formation of the final estimates of the CR-levels according to the established comparison test at the scale of the segment $[0; 100]$. The fuzzy maxmin convolution method, which is the essence of the second approach, solves the problem by using another way of aggregating of expert judgements of the CR-factors. A comparison of summarized results of CR-levels estimating of hypothetical alternatives (countries) a_t ($t = 1 \div 10$) obtained by both methods is presented in Table 4, which shows that the orders of some estimates of the CR-levels do not coincide.

Acknowledgement. The authors consider necessary to express their sincere appreciation to Professor R.R. Rzayev for the help that he rendered during the process of writing and preparing this article.

References

1. PwC Global Risk podcast series: <https://www.pwc.com/gx/en/services/advisory/consulting/risk/cosoerm-framework/podcasts.html>. Accessed 15 May 2018
2. Lin, A.S.: A note on the concordance correlation coefficient. *Biometrics* **56**, 324–325 (2012)
3. Lin, A.S., Wu, W.: *Statistical Tools for Measuring Agreement*. Springer, New York (2012). <https://doi.org/10.1007/978-1-4614-0562-7>

4. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning. *Inf. Sci.* 8(3), 199–249 (1965)
5. Rzayev, R.R.: *Analytical Support for Decision-Making in Organizational Systems*. Palmerium Academic Publishing, Saarbruchen (2016). (in Russian)